Surviving a High-Rise Fall

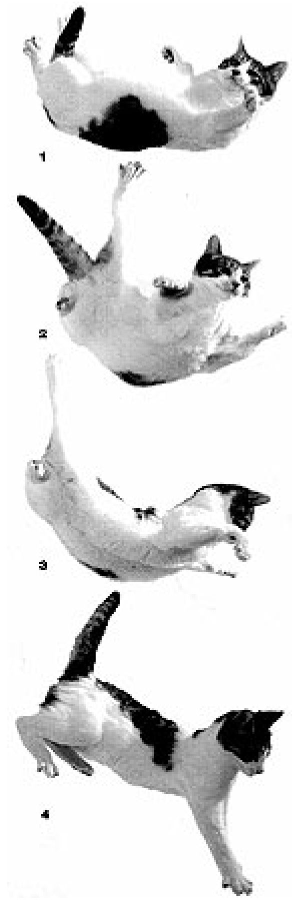


Figure 1: http://www.physlink.com/education/askexperts/Images/ae411a.jpg

Introduction

If an object falls freely to the ground, we say it undergoes a free fall. In physics however, free-fall has a strict meaning, that is, the only force acting on the object is its gravity. Near the surface of the earth, gravity is constant, therefore a free-falling object has a constant acceleration, $a_y = g$ (choosing downward to be positive), and velocity increase linearly with time

$$v_y = v_{0y} + gt \qquad (1)$$

Eq. (1) can explain the motion of a rock falling to the ground pretty well, but why does a piece of paper fall more slowly than a rock? Why can a cat survive a 10-story fall but a human cannot?

The answer is, of course, is that the air resistance was neglected in Eq. (1). If the effect of the air is not negligible, then the acceleration is not constant, and the motion becomes complicated, as we will soon see.

Applying Newton's 2^{nd} law to an object falling downwards in the air, taking down as positive, we have

$$mg - F_d = ma_y$$

where F_d is the *drag force*, the force due to air resistance. Note that it points upwards, opposite the direction of the motion.

Unlike gravity, the drag force is not constant: it depends on the velocity. A simple yet effective approximation for the drag force for a rigid object, moving at relatively slow speeds is

$$F_d = bv^2$$

where b is a constant for a given object. Then Newton's 2^{nd} law can be written (dropping subscripts – we are only considering motion in one dimension)

$$mg - bv^2 = ma$$

or, rearranging,

$$a = g - \frac{b}{m}v^2 \qquad (2)$$

As you see in Equation (2), the acceleration depends on velocity, which is changing depending on acceleration. So it is hard to solve Equation (2) analytically. However, we can solve it numerically with the power of computer programs.

Our goal is to solve for the motion, that is, the position, the velocity and the acceleration at any given time. To solve the problem numerically, we need to make some assumptions. We first slice the time into many small steps Δt. We assume that the velocity is constant within each step Δt, and increases by $\Delta v = a\Delta t$ at the end of each step, as shown by the heavy short lines in Figure 2.

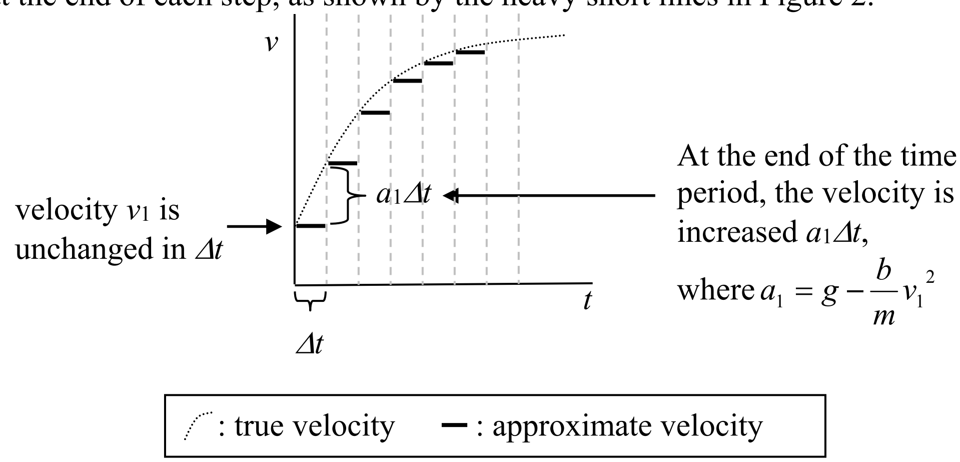
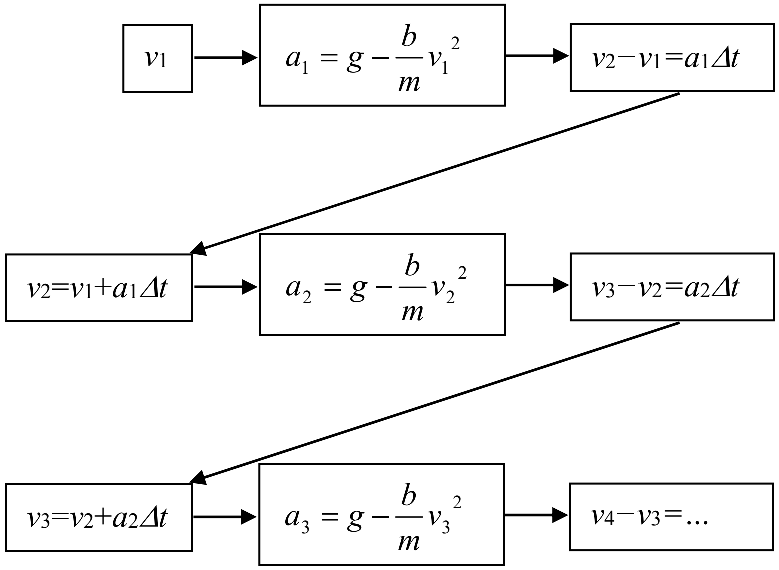


Figure 2

So, if we know the velocity in the first time step v_1, we can use Eq. (2) to calculate the acceleration in the first step a_1, which tells us how much the velocity changes at the end of step 1. That gives us the velocity in the second time step v_2. Repeating the process we can find the velocity in the third time step v_3, and so on. This numerical method is called Euler's Method.



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From Figure 2, one can see that as we choose smaller Δt, the heavy short lines will merge into the actual motion—the dotted curve. While this involves a huge amount of repetitive calculations, it can be easily done by a computer program, in particular a spreadsheet program like MS Excel.

We need to tell Excel the value of *b/m*. The parameter *b* describes the size of the drag force in $F_d = bv^2$, which depends on the cross section area, the shape and material of the object.

Compare a human skydiver and a cat. Who do you think will have a larger *b* value? How about *b/m*? The *b/m* values will be given in the lab.

In order to get prepared for the lab, you may also want to read "How to Use Excel".