Archimedes' Principle

Purpose

Problem 1: use Archimedes' Principle to find the density of a metal cylinder. Problem 2: use the same method to find the density of a rock of irregular shape.

Introduction and Theory

When a solid object is submerged in liquid, it experiences a vertical upward force. This force is called the buoyant force. Archimedes' principle states that the buoyant force equals the weight of the liquid displaced by the solid object:

*F*_{buoyant} = *W*_{displaced liquid}

Archimedes principle applies to gas too. However, as the weight of the gas displaced by the object is usually very small, we can ignore the buoyant force from the gas, and call the weight measured in air its true weight W. If we submerge the object in a liquid and measure the weight W' (known as "apparent weight"), we can find the density of the solid:

The weight of the solid in air is: $W = mg = \rho Vg$ where ρ and V are the density and the volume of the solid object.

The buoyant force when submerged in liquid is: $W - W' = W_{displaced liquid} = \rho_{liquid} Vg$ where ρ_{liquid} is the density of the liquid, which is assumed to be already known.

Take the ratio of two equations above, we get

$$rac{
ho}{
ho_{ ext{liquid}}} = rac{W}{W-W'}$$
 , therefore, $ho = rac{W}{W-W'}
ho_{ ext{liquid}}$

That is, one can measure the density of the solid object from the weight and the apparent weight, which can be measured directly using a spring scale. This way of determining density is especially useful when the solid has an irregular shape but has no hollowness, like a crown. The legend was that the King of Syracuse asked Archimedes to find out whether his crown was made of pure gold. Archimedes came to the idea of using buoyant force while taking a bath. He was so excited and ran to the streets naked, yelling "Eureka!" ("I have found it!") He was then able to show that the crown was not made of pure gold.

Because the weight equals mass times gravity acceleration *g*, we can use masses instead of weights, as masses can be measured more precisely with a 2-pan balance, as shown in Fig. 1:

$$\frac{\rho}{\rho_l} = \frac{W}{W - W'} = \frac{mg}{mg - m'g} = \frac{m}{m - m'}, \text{ therefore, } \rho = \frac{m}{m - m'}\rho_l$$

If the density of the liquid ρ_l is known, all we need to measure are the mass *m* and the "apparent mass" *m*'. The buoyant force of the air is ignored as before.

find mass of a solid *m*

find "apparent mass" of a solid *m*' in water



Fig 1

Archimedes' Principle – Prelab

Print this page and finish the questions before coming to the lab. You will not be given a cylinder and rock until this assignment is completed correctly. Do not hand in this page.

1. Given $\rho = \frac{m}{m - m'} \rho_l$, derive the equation to calculate the relative uncertainty of the rock $\frac{\delta \rho}{\rho}$.

2. A student measured a rock has a mass of (1000.00 ± 0.15) g in air, and an "apparent mass" of (600.00 ± 0.20) g when the rock is submerged in tap water. The density of tap water is $(1.00 \pm 0.01) \times 10^3 \text{ kg/m}^3$.

Calculate the density of the rock and its uncertainty. Round off to the correct digits.

Problem 1 Density of a Metal Cylinder

A template of lab report for Problem 1 is provided at the end of this lab manual.

Each partner must measure a different metal cylinder for this problem.

List all the apparatus in the "Apparatus" section, with the identifying number for the cylinder and the 2-pan balance. Record the colour of the cylinder too. You may also sketch the setup to show how the cylinder is suspended under the 2-pan balance, but there is no need to draw the details like in Fig. 1.

To take data, first hang the cylinder under the 2-pan balance with a thread, looping it on the metal hook right under the *centre* of the left pan. See Fig 1. Measure and record the mass of the cylinder in air *m*. There is uncertainty when reading the 2-pan balance, plus each counter mass will contribute 0.05 g to the uncertainty.

Then, place the beaker of water under the balance so that the cylinder is completely submerged in the water. You may have to raise the beaker with a book or the mass box. See Fig. 1. Record the balance reading while the cylinder is in water m' and its uncertainty.

Calculate the density of the cylinder ρ under "Calculations". Do not round off the result yet. Calculate the uncertainty under "Uncertainty Analysis": relative uncertainty ($\delta \rho / \rho$) first, then the absolute uncertainty $\delta \rho$. State the result in "Conclusions" in the proper format and finish the report with the "Discussions". The reference values are listed below:

. . .

Reference	values for	Density	or the	Cylinder:	

Metal	Density (\times 10 ³ kg/m ³)		
Aluminium	2.70 ± 0.01		
Brass	8.56 ± 0.01		
Copper	8.89 ± 0.01		
Steel	7.85 ± 0.01		

Problem 2 Density of a Rock

Partners may share the same data for this problem.

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Use the same method as in Problem 1 to measure the density of a brecciated jasper rock, with a reference density of (2.63 to 2.70) \times 10³ kg/m³. Write the report on your own paper, following the format in the template.

After all the data are collected, drain the water out of the beaker. DO NOT throw away the threads after use!

Name:	-
Partner(s):	_
Desk:	_
Date:	

Measuring Densities using Archimedes' Principle

Problem 1: Density of a Cylinder

Purpose: Use Archimedes' Principle to measure the density of a metal cylinder

Apparatus:

Data:

Mass of Cylinder # _____ in air and in water (in grams)

	Reading	# of counter masses	Uncertainty
Mass in Air (m)			
"Apparent Mass" in Water (m')			

Density of water (ρ_w): (1.00 ± 0.01) × 10³ kg/m³ (given by the instructor)

Calculations:

Uncertainty Analysis:

Conclusions:

The density of cylinder #____ was measured to be (_____ \pm ____) × 10³ kg/m³ (±___%), using Archimedes' Principle.

Discussions:

As shown by the comparison graph below, the density of our cylinder #_____ is ______

We conclude that this cylinder is made of _____.

Possible systematic effects include: