Name: ________________________________
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Date: _________________

Graphing and Simple Harmonic Motion, Part I

Many Physics concepts can be modeled as linear relationships: Newton's 2nd Law ($F = ma$), Hooke's Law ($F = k\,\Delta x$), Ohm's Law ($V = I\,R$), etc. When experimental data is collected to test these relationships, physically meaningful constants can be found (mass, spring constant, resistance) from the slope of a graph of the data. This is the basis of the $y = mx+b$ form of a linear equation.

However, not all relationships are linear. In fact, even some "linear" relationships are only linear in a limited range (e.g. not all conductors follow Ohm's Law at very low or very high currents). Examples of common non-linear Physics and Biology include:

- exponential population growth equations: $P = P_o e^{rt}$
- intensity - distance equations (aka inverse square law): $I = P/4\pi r^2$, $F = GM_1M_2/r^2$ etc.
- oscillatory motion: $x = A\sin(\omega t+\theta)$
- kinematics with constant acceleration: $y = y_o + v_o t + 1/2at^2$
- volume of a sphere: $V = 4/3\pi r^3 = 1/6\pi d^3$

We can plot experimental data for all of these phenomena, but the graphs would NOT be straight lines. Thus a graph would not be all that useful as a calculation tool, but only as a way of visually seeing the relationships as a picture.

In this worksheet/lab, we will work through some ways of analyzing experimental data with graphs, both when a relationship DOES follow a linear equation, and when it might not.

Problem 1: Hooke's Law

Springs are tightly wound coils of wire with a structure that will store potential mechanical energy. When compressed or stretched, the force it exerts is proportional to its change in length. This property of springs is called "Hooke's Law", after the guy who experimentally discovered it.

The equation for Hooke's Law is: $F = k\,\Delta x$,

where F is the force, either the spring pushing out (for compression) or pulling in (for extension),
Δx is the compression or extension length from the spring's rest position
and k is the proportionality constant, called the "spring constant" or "force constant", and has units of force/length, which would be Newtons/meter (N/m) in SI units.

Since the form of Hooke's Law is in the linear form of $y = slope \bullet x+b$, it is easy to recognize that the value of the force constant can be found by plotting force on the vertical axis and extension on the horizontal axis of a graph, and calculating the slope of the resulting best-fit line.

Apparatus:

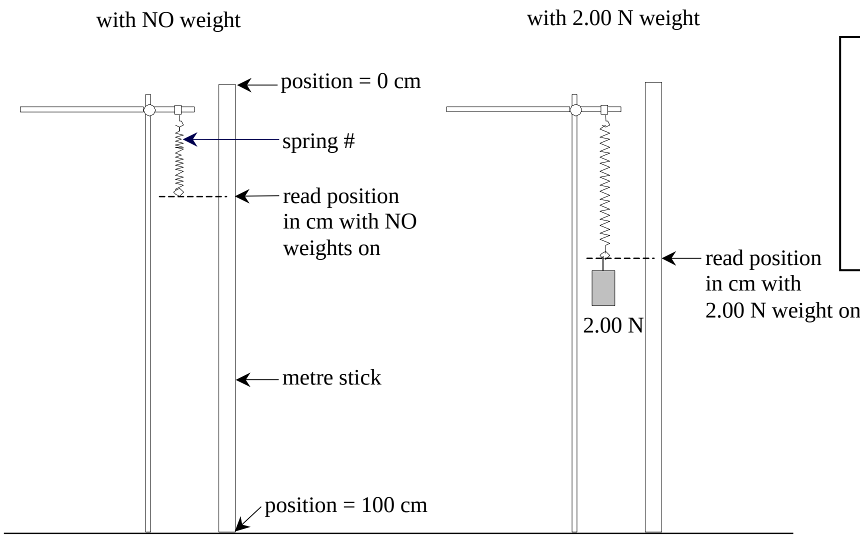

Fig 1

Spring # ____

Reference value for spring constant for spring # ____
_____ ± 0.1_N/m_

Data:

Weight F /N (applied force)	Position x /cm	Extension Δx /cm
0.00 ± 0.00	±	
2.00 ± 0.05	±	±
5.00 ± 0.05	±	±
7.00 ±	±	±
9.00 ±	±	±
12.00 ±	±	±
15.00 ±	±	±

Calculations: Draw the graph.

- As you plot your data points, include their uncertainties with error bars.
- Draw your "best fit" and "worst fit" lines.
- The "best fit" line should pass as close as possible to as many of the data points as possible. (It does not necessarily HAVE to go through ANY of the points, nor the origin!)
- The "worst fit" line is the line that has a slope that is as different as possible from the "best fit" line, but still touches all of the boxes formed by the horizontal and vertical error bars.
- Calculate the slopes of your "best fit" and "worst fit" lines, on the graph, using big slope triangles and NOT using data points.
- Remember your units, and a zillion other details. (see Graphing: Advanced document)

See graph "Using Hooke's Law to find spring constant for spring # ____" for slope calculations.

$$slope_{best} = \frac{rise}{run} = \frac{F_{b2} - F_{b1}}{\Delta x_{b2} - \Delta x_{b1}}$$

$$= \frac{(\qquad - \qquad)N}{(\qquad - \qquad)cm}$$

$$= \qquad \frac{N}{m}$$

(note: These calculations are to be done ON THE GRAPH. You can summarize them in your report too if you like.)

Uncertainty Analysis:

$$slope_{worst} = \frac{rise}{run} = \frac{F_{w2} - F_{w1}}{\Delta x_{w2} - \Delta x_{w1}}$$

$$= \frac{(\qquad - \qquad)N}{(\qquad - \qquad)cm}$$

$$= \qquad \frac{N}{m}$$

$$\delta slope = slope_{best} - slope_{worst}$$

$$=$$

$$= \qquad \frac{N}{m}$$

so: $k =$ (do not round anything yet!!!)

$$\delta k = \delta slope = \qquad \frac{N}{m}$$

$$\% \delta k = \frac{\delta k}{k} \times 100\%$$

$$= \frac{\frac{N}{m}}{\frac{N}{m}} \times 100\%$$

$$= \qquad \%$$

$$\approx \qquad \%$$

(note: These calculations would be done in your report.)

Conclusions (now you do appropriate rounding)**:**

The spring constant for spring # ______ was found to be _(_______±_____)_ *N/m* (±____%) , using Hooke's Law. The reference value for spring # ______ is _(_______±_____)_ *N/m*, according to the list in T346 or on its tag.

Discussion:

$$discrepancy = \frac{|ref - calc|}{ref} \times 100\%$$

$$= \frac{(\qquad - \qquad)}{(\qquad)} \times 100\%$$

$$= \qquad\qquad \%$$

$$\approx \quad \%$$

N/m

Spring constant #

Since the range of the measured and reference values ________ overlap, our values _______ agree.

Other comments and observations: (examples: Does Hooke's Law always apply? How can we make the experiment "better"? Is there a y-intercept on the graph, and what does it mean?)

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Prelab for Problem 3: Complete these pages before the next lab session

Problem 2: Creating Linear Graphs from a Non-Linear Function

Now that the graphing and analyzing of linear relationships is fairly clear, we will extend the process to use graphs to analyze other experimental data that are NOT linear.

See the online document Graphing: Advanced for another version of these instructions.

The first part of the process is to look at the theoretical equation that the experiment is supposed to verify. From its structure, it may be possible to change the variables in the equation to TURN IT INTO A LINEAR EQUATION that would then be graphed as in Problem 1.

The period, T, of a mass, m, oscillating on a spring, depends on the spring constant, k. By measuring the period for different masses, the spring constant can be found.

$$T = 2\pi\sqrt{\frac{m}{k}}$$

In this equation, the period of oscillation can be measured for given/known masses. Plotting T *vs* m, though, would NOT give a straight line. In order to turn this equation into the linear form of $y = slope \bullet x + b$, we can keep T as the y variable, and

if we assign $x =$, the equation will become

The equation has now become linear! The slope of the linearized graph would now be

slope =

and thus

$k =$

The change of the x variable will require us to calculate all of the new x values, and the uncertainty of x, δx, will have to be calculated too.

Raw data: T (period), m (mass)	Desired quantity: k (spring constant)
Graphing data: x-axis: , y-axis:	Find the desired quantity from the slope of the graph:
Uncertainty for graphing data: $\delta x =$, $\delta y =$	Find the uncertainty of the desired quantity:

You may use the blank area below for scratch work:

Let's practice this change-of-variable process on a few more equations that aren't linear.

1..The mass, m, of a sphere depends on the density, ρ, of the material and the diameter, d. By measuring the mass and diameter of several spheres of the same material, the density of the material can be calculated.

$$m = \frac{\rho \pi d^3}{6}$$

Raw data:	Desired quantity:
Graphing data: x-axis: , y-axis:	Find the desired quantity from the slope of the graph:
Uncertainty for graphing data: $\delta x =$, $\delta y =$	Find the uncertainty of the desired quantity:

2. The intensity I of a point source of light depends on the distance, r, from the light source. By measuring the intensity at various distances, the power output, P, of the light source can be found.

$$I = \frac{P}{4\pi r^2}$$

Raw data:	Desired quantity:
Graphing data: x-axis: , y-axis:	Find the desired quantity from the slope of the graph:
Uncertainty for graphing data: $\delta x =$, $\delta y =$	Find the uncertainty of the desired quantity:

3. For a given length of string, L, a standing wave can be set up on the string. By measuring the frequencies of these standing waves, the velocity of the wave, v, can be found.

$$f_n = \frac{nv}{2L}$$

(Hint: The harmonic number n has no uncertainty. The string length L has an uncertainty δL.)

Raw data: Harmonic number: n (n=1, 2, 3…) Frequencies: f_n	Desired quantity:
Graphing data: x-axis: , y-axis:	Find the desired quantity from the slope of the graph:
Uncertainty for graphing data: $\delta x =$, $\delta y =$	Find the uncertainty of the desired quantity:

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Graphing and Simple Harmonic Motion, Part II

Problem 3: Simple Harmonic Motion

Recap:
In Problems 1 and 2, we discovered that we could take a theoretical relationship between measurable variables and calculate something useful from a graph. In order to use a graph's slope, a relationship needs to be linear ($y = slope \bullet x + b$), but if the measured quantities do NOT have a linear relationship, a graph isn't all that helpful as a calculation tool. However, we figured out a way of manipulating the variables to turn the theoretical relationship into a linear one, thus letting us use the slope of its graph to calculate/measure something physically meaningful.

The period, T, of a mass, m, oscillating on a spring, depends on the spring constant, k. By measuring the period for different masses, the spring constant can be found.

$$T = 2\pi\sqrt{\frac{m}{k}}$$

Raw data:	Desired quantity:
Graphing data: x-axis: , y-axis:	Find the desired quantity from the slope of the graph:
Uncertainty for graphing data: $\delta x =$, $\delta y =$	Find the uncertainty of the desired quantity:

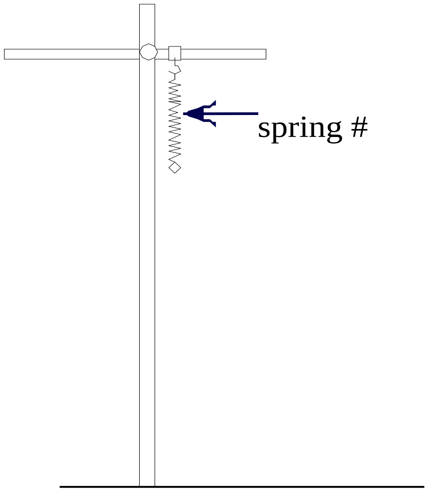
In this experiment, we will measure the period of oscillation by using a stopwatch to get the time for 20 oscillations, t_{20}. From that value, the average period and its uncertainty would be found.

$T = t_{20} / 20$

Now we can assume that δt_{20} is based on your reaction time (ask how you can test this with a stopwatch) and that $\delta T = \delta t_{20}/20$

After all of the data has been collected and the data points for plotting have been figured out, you are ready to graph! And then you can calculate the spring constant.

Apparatus:



stopwatch #

Spring # ____

Reference value for spring constant for spring # ____ ___________N/m_

Data:

Time for 20 bounces t_{20} /s	Period T /s	δT /s	Mass m /kg	δm /kg	x =	δx =
			0.500			
			1.500			

Calculations: Plot a graph of your *y vs x* values from the data table.

On the graph, in ink, calculate the following:

- The slope of your "best fit" line: slope_{best} (or the slope)
- The slope of your "worst fit" line: slope_{worst}
- The uncertainty of the slope: $\delta\text{slope} = \left|\text{slope}_{best} - \text{slope}_{worst}\right|$

Calculate the spring constant from the graph's best fit slope:

k = (don't round anything yet!!!!!!)

Uncertainty Analysis:

From the uncertainty of the slope, find the relative (and %) and absolute uncertainties of the spring constant.

Conclusions (in proper format):

The spring constant of spring # _____, was measured to be ______________________________.using simple harmonic motion. The reference value from the list in T346 is ______________________.

Discussions: (discrepancy, do the values agree, compare to the results from Problem 1, other comments)