Archimedes' Principle

Purpose

Problem 1: use Archimedes' Principle to find the density of a twig.
Problem 2: use Archimedes' Principle to find the density of a rock.

Introduction and Theory

When a solid object is submerged in liquid, it experiences a vertical upward force. This force is called the buoyant force. Archimedes' principle states that the buoyant force equals the weight of the liquid displaced by the solid object:

$$F_{\text{buoyant}} = W_{\text{displaced liquid}} \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots. (1)$$

Archimedes principle applies to gas too. However, as the weight of the air displaced by the object is usually very small, we can ignore the buoyant force from the air.

For objects less dense than water

If the object floats in the liquid, the weight and the buoyant force obviously balance: (Draw a FBD to yourself.)

$$W_{\text{solid}} = W_{\text{displaced liquid}}$$

Because weight equals mass times gravitational acceleration, and mass equals density times volume, above equation can be written as (the symbol for density is ρ):

$$(\rho_{\text{solid}} V_{\text{solid}})g = \left(\rho_{\text{liquid}} V_{\text{displaced liquid}}\right)g$$

Therefore,

$$\rho_{\text{solid}} = \frac{V_{\text{displaced liquid}}}{V_{\text{solid}}}\rho_{\text{liquid}} = \frac{V_{\text{submerged}}}{V_{\text{total}}}\rho_{\text{liquid}} \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots. (2)$$

Above equation tells us, if the density of the liquid is known, we can tell the density of the solid from how much volume is submerged in the liquid. Using water as the liquid is particularly convenient, not only because water is easily available, but also because the density of water = 1.00 g/cm^3. For example, if you see a styrofoam floating in water with about 10% of its volume inside the water, you immediately know the density of the styrofoam is 0.1 g/cm^3. In the lab, we will use this method to measure the density of a twig.

For objects more dense than water

If the solid object is denser than water, the situation is a little bit more complicated. In order to find the density of the solid, we need to know how much weight is "lost":

Weight lost = Weight (W) – Apparent weight (W') = Buoyant force (F_{buoyant})

If the solid is entirely submerged in the liquid, above equation can be written as

$$W - W' = F_{\text{buoyant}} = W_{\text{displaced liquid}} = (\rho_{\text{liquid}} V_{\text{solid}})g$$

Comparing this equation to $W = (\rho_{\text{solid}} V_{\text{solid}})g$, we get

$$\frac{\rho_{\text{solid}}}{\rho_{\text{liquid}}} = \frac{W}{W - W'} \quad \text{or} \quad \rho_{\text{solid}} = \frac{W}{W - W'}\rho_{\text{liquid}}$$

Therefore, we can measure the density of a solid object from the weight and the apparent weight in the liquid with known density. This way of determining density is especially useful when the solid has an irregular shape but has no hollowness, like a crown. The legend was that the King of Syracuse asked Archimedes to find out whether his crown was made of pure gold. Archimedes came to the idea of using buoyant force while taking a bath. He was so excited and ran to the streets naked, yelling "Eureka!" ("I have found it!") He was then able to show that the crown was not made of pure gold.

In the lab, we will measure the masses (Fig. 1) rather than weights, as the 2-pan balance gives more sig. figs. than the spring scale. The weights are proportional to masses by the gravity acceleration g, so

$$\rho_{\text{solid}} = \frac{W}{W - W'}\rho_{\text{liquid}} = \frac{mg}{mg - m'g}\rho_{\text{liquid}} = \frac{m}{m - m'}\rho_{\text{liquid}} \quad \text{(3)}$$

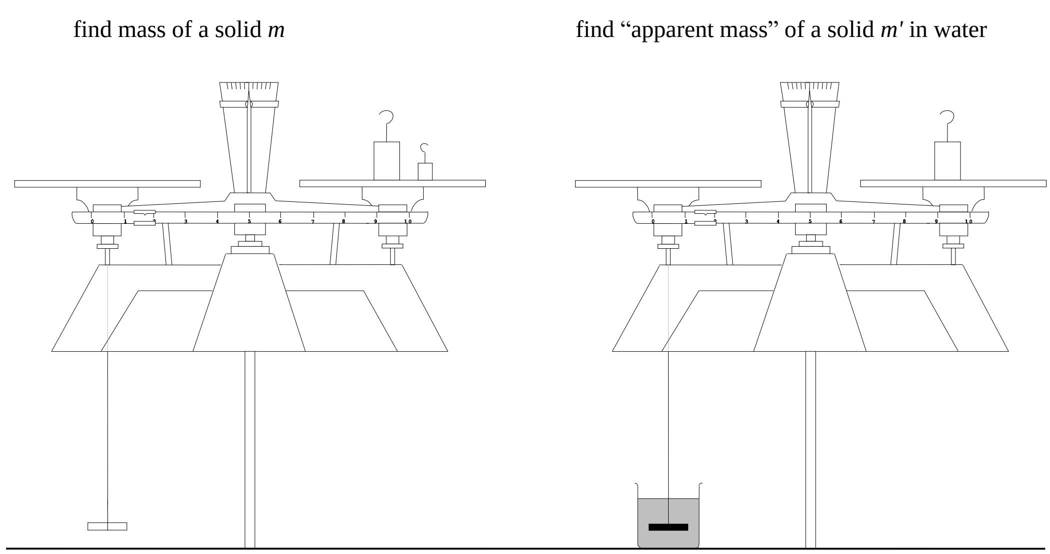

Fig 1

Problem 1 Density of a Twig

Give the "Purpose" first, then list all the apparatus in the "Apparatus" section with identifying numbers.

Fill up the long plastic cylinder with water up to about 5cm below the top. Place the twig in the cylinder and let it stabilize. Record the total length of the twig and the length under the surface of the water, using the division marks on the twig. We can also call them the total volume and the submerged volume, assuming the twigs have uniform thickness. Because we only need the ratio between the two volumes, the exact units do not matter.

The total length or volume is 10.0 divisions with negligible uncertainty, and the part under the water has an uncertainty of a fraction of one division that you should decide when taking the reading.

Use Eq. (2) and $\rho_{\text{water}} = 1.00 \times 10^3$ kg/m^3 to calculate the density of the twig, then state the result with proper sig. figs. The reference densities for the twig vary from 600 to 900 kg/m^3 so you do not need to calculate the percentage discrepancy. Instead, state whether your result is within the reference range.

Problem 2 Density of a Rock

As usual, start with the "Purpose" and the "Apparatus" sections. To take data, first hang the rock under the 2-pan balance with the thread, looping it on the metal hook right under the *centre* of the left pan. See the left diagram of Fig 1. Measure and record the mass of the rock in air m, together with its uncertainty. Then place the beaker of water under the balance so that the rock is completely submerged in the water as shown by the right diagram of Fig. 1. You may have to raise the beaker with a book or the mass box. Record the "apparent mass" m' and its uncertainty.

Calculate the density of the rock using Eq. (3) and $\rho_{\text{water}} = 1.00 \times 10^3$ kg/m^3, and then calculate the percentage discrepancy of your result to the density of a brecciated jasper rock, 2.67×10^3 kg/m^3. State the result in the correct format.

Question (Answer it at the end of the report.)

How does this method of measuring the densities compare to the "Density" lab?

After finishing the lab report, drain the water out of the plastic container/beaker and tidy up your desk.