## Energy

Conservation of energy is a universal law, but it can take different forms. In this lab, you will investigate two cases of energy conservation of a cart rolling down an inclined plane. In one case, Problem 1, frictional forces will be considered negligible so that mechanical energy will be conserved. In the other case, Problem 2, the external forces doing work on the cart will not be negligible, so that conservation of mechanical energy will not hold.

Note: You will write a SEPARATE lab report for each problem.
The following notation is used in Problem 1 and/or Problem 2:

- KE: Kinetic energy of the cart.
- GPE: Gravitational potential energy of the cart (*actually, it's the potential energy of the cart and earth system).
- $M E=G P E+K E$ : Mechanical energy, the sum of the kinetic and potential energies.
- $E_{\text {thermal }}$ : Thermal energy.


## Problem 1 Conservation of Mechanical Energy

## Purposes

Verify the conservation of mechanical energy by letting a low-friction cart down an inclined plane.

## Introduction and Theory

In this problem, a low-friction cart rolls down a track. While the cart is coming down, the gravitational potential energy (GPE) constantly turns into kinetic energy ( $K E$ ) of the car. Ignoring friction, the only force that does work is gravity, which is a conservative force, so the mechanical energy $(M E)$ of the cart-earth system is conserved. That is, the sum $M E=G P E+K E$ remains unchanged. This is equivalent to saying that the change in the mechanical energy is zero, or $\Delta M E=0$. You will verify this principle of energy conservation in this problem.

Figure 1 above illustrates the apparatus setup. Initially, the cart is at rest at the "top of the hill" ( 100 cm mark) and has only GPE. We let the cart go down the hill frictionlessly and measure its speed at the "bottom of the hill" ( 30 cm mark). The cart has both GPE and $K E$ at the bottom of the hill (See Figure 1). We will calculate the initial and final $M E$ to verify its conservation.


Figure 1

The following additional notations are used for Problem 1 (See Figure 1):
$m=$ mass of cart, including the picket fence.
$g=$ acceleration due to gravity $=9.81 \mathrm{~m} / \mathrm{s}^{2}$
$h_{\text {initial }}=$ the height at the top of the hill.
$h_{\text {final }}=$ the height at the bottom of the hill.
$v=$ the speed of the cart at the bottom of the hill, measured by a photogate.

## Apparatus

Draw Figure 1, including labels and identifying numbers. List any other apparatus used but not shown. Write their identifying numbers as well.

## Data

1. Measure and record $m$, the mass of the cart including the picket fence. Remember the uncertainty of mass measurements will increase by 0.05 g for every counter mass that you use.
2. To find the GPE, we need to find the height at the "top to bottom" and at the "bottom of the hill". This is a bit tricky because our desktop may not be leveled. To deal with this problem, let us level the track first. Lay the track flat on the desktop and put the cart on the track. Adjust the supporting screw until the cart can stay still. Give the stationary cart a gentle push to the left then to the right. If the track is leveled, the cart would slow down to a stop in a similar way. If not, adjust the supporting screw again until the track is leveled.
3. Lay the 0 cm end of the meter stick perpendicular against the desktop and measure the height of the track at the 100 cm mark and 30 cm mark: If they are the same, your desktop is leveled. You can cross out the first row of Table 1, and write next to it "The desktop is leveled." If they are different by 0.1 cm or more, you must record the two height readings in the first row of Table 1, together with the uncertainty.
4. Do not move the 0 cm end of the track and raise the 120 cm end using three wooden blocks, as shown in Figure 1. Again, measure and record the height of the track from the desktop at both marks and record them in the second row of Table 1. The uncertainties should not change.
5. Calculate the height of the top and of the bottom of the hill $h_{\text {initial }}$ and $h_{\text {final }}$ (see Figure 1 ) and record them in the third row. These are the heights measured from a leveled surface, either from the desktop, or from the top of the track.

Table 1: measurements of the heights ( cm )

|  | Top of the hill <br> $(100 \mathrm{~cm}$ mark $)$ | Bottom of the hill <br> $(30 \mathrm{~cm}$ mark $)$ | Uncertainty |
| :--- | :--- | :--- | :--- |
| When the track is leveled |  |  |  |
| When the track is raised |  |  |  |
| Height of the marks | $h_{\text {initial }}=$ | $h_{\text {final }}=$ |  |

6. To find the $K E$ at the "bottom of the hill", we need to measure the speed. Connect the photogate to the DIG/SONIC 1 port of the LabPro interface box. Mount the photogate on the track so it is centered at the 30 cm mark. Adjust the arm of the photogate so that the light sensor is at the same height as the 5 cm band of the picket fence.
7. Start Logger Pro and open file $\backslash$ Probes \& Sensors $\backslash$ Photogates $\backslash$ One Gate Timer.cmbl. Change the
photogate distance to 0.050 m . Logger Pro will measure the time and use "distance over time" to find the speed.
8. Line up the middle the cart with the 100 cm mark and hold it stationary. Click the "Collect" button in Logger Pro. After you hear the clicking sound, let the cart go. The Logger Pro program will display the speed of the cart when it passes the photogate. Repeat this process to get two measurements of the speed and list them in a data table. The data table should have a title, units, two readings of the speed, the average of two speeds and the difference between two speeds. We will use the average speed to calculate $K E$, and the difference as the uncertainty in the speed.

## Calculations

Before calculations, check that you have all the required data: the mass, two heights and two speeds. Keep the track setup for Problem 2.

First, convert the mass into kilograms and the heights into meters. Then, calculate $K E, G P E$ and $M E$ at the initial position ( 100 cm mark). And finally, calculate $K E, G P E$ and $M E$ at the final position ( 30 cm mark). The equations to calculate the energies are: [Remember to start all calculations with the corresponding symbolic equation.]

$$
K E=\frac{1}{2} m v^{2}, \quad G P E=m g h, \quad M E=K E+G P E
$$

Is $M E$ conserved? Let's calculate its change " $\triangle M E=$ final $M E-$ initial $M E$ ". If the size of $\triangle M E$ is less than its uncertainty, we can say $M E$ is conserved. In the higher level lab courses, we will learn how to calculate the uncertainty of $\triangle M E$. For now, we will simply take the uncertainty to be $2 m g \delta h$, where $\delta h$ is the uncertainty of the height measurement. There is a factor of 2 because there are two heights related.

Calculate $\Delta M E=$ final $M E-$ initial $M E$ and $2 m g \delta h$ in your report.

## Conclusions

In sentence form, state your results of "initial $M E$ " and "final $M E$ ", to the correct sig. figs. Then state the change in $M E$ (the absolute value). Conclude whether $M E$ is conserved for the cart going down the track, based on the comparison of the change in $M E$ and its uncertainty, $2 m g \delta h$.

## Problem 2 Work and Energy

## Purpose

Use the relationship between work and energy to determine the force of friction when pulling a loaded cart up an inclined plane.

## Introduction and Theory

Problem 2 is based on the relationship between the work done on the system ( $W$ ) and the change in the system's energy ( $\triangle K E, \triangle G P E$ and $\Delta E_{\text {thermal }}$ ). We will use a spring scale to pull a loaded cart slowly up the same track used in Problem 1, from the $30-\mathrm{cm}$ mark to the $100-\mathrm{cm}$ mark. This time, we will flip the cart upside down, so friction cannot be ignored. As a result, mechanical energy is not conserved, and it changes by an amount equal to the work done on the system by the external forces. In our case there is only one external force, namely the pulling force. The energy of the system will change by the work done on the system by the external forces, in our case it is the pulling force:

$$
\Delta K E+\triangle G P E+\Delta E_{\text {thermal }}=\text { Work done by the pulling force } .
$$

We will not test this principle in the lab. Instead, we will assume it is true, and use it to calculate first $\Delta E_{\text {thermal }}$, then the force of friction.

The following additional notations are used in problem 2:
$m=$ mass of cart, including the frictional block.
$g=$ acceleration due to gravity $=9.81 \mathrm{~m} / \mathrm{s}^{2}$
$h_{\text {initial }}=$ the height at the top of the hill.
$h_{\text {final }}=$ the height at the bottom of the hill.
$L=$ distance travelled from the 30 cm mark to the 100 cm mark, $L=70.0 \mathrm{~cm}$.
$F=$ the average force pulling the cart up the track, as read from the spring scale.
$f_{k}=$ kinetic frictional force acting on cart.

## Apparatus

The setup is similar to that of Problem 1, with the exception of these changes: the picket fence is replaced by the friction block and the photogate is replaced by a spring scale. Draw the new setup, and list and identify the apparatus.

## Data

1. Measure and record $m$, the mass of the cart together with the friction block.
2. Place the friction block into the cart, with the felt side facing out, then flip the cart upside down and place it on the track. Using the spring scale, pull the cart as slowly and steadily as possible, from the 30 cm mark to the 100 cm mark, keeping the spring scale parallel to the track. Record the reading of the spring scale with uncertainty: its fluctuation or uncertainty should not be bigger than half of the smallest division, otherwise you are not pulling it steadily enough.
3. The heights are the same as Problem 1, so we need not to measure them again. Note, however, that the definition of "initial" and "final" have been reversed: initial $\rightarrow$ the cart is at $30-\mathrm{cm}$ mark; final $\rightarrow$ the cart is at $100-\mathrm{cm}$ mark.
4. Lay the track flat and pull the cart on the track using the spring scale slowly and steadily. Write down the force read from the scale. Denote this force as $F_{\text {flat }}$. [You will use this result in your conclusion.]

## Calculations

First, convert the mass into kilograms. Copy $h_{\text {initial }}$ and $h_{\text {final }}$, the top and bottom heights in meters from Problem 1. Convert the distance $L=70.0 \mathrm{~cm}$ into meters.

The work done by the pulling force is $W_{\mathrm{F}}=F L \cos \theta$, where $\theta$ is the angle between the force and the displacement. Calculate the work $W_{\mathrm{F}}$ done by the pulling force as the cart moves from 30.0 cm to 100.0 cm .

Calculate the change in the kinetic energy, $\Delta K E=K E_{\text {final }}-K E_{\text {initial }}$, as you pull the cart up the incline.

Calculate the change in the gravitational potential energy, $\triangle G P E$, as you pull the cart up the incline.
Use the relation between work and energy, $\triangle K E+\triangle G P E+\Delta E_{\text {thermal }}=W_{\text {pull }}$ to find the change in the thermal energy $\Delta E_{\text {thermal }}$.

Based on $\Delta E_{\text {thermal }}=f_{k} L$, calculate the magnitude of the kinetic friction force $f_{k}$ acting on the cart as it was pulled up the plane.

## Conclusions

Report the force of the kinetic friction on the cart to the correct number of significant figures.
Check your result:
Based on Newton's second law, when the track is flat, the pulling force $F_{\text {flat }}$ equals to the kinetic friction. Your calculated friction should be roughly the same. (In theory, the friction on a $5^{\circ}$ slanted track should be smaller than the friction on the flat track by $0.4 \%$.)

Comment on your results. How well do they agree/disagree with this statement?

