## Forces and Equilibrium

## Purpose

Verify the equilibrium condition of concurrent forces.

## Introduction and Theory

When an object is in static equilibrium, Newton's First Law states that the vector sum (or resultant) of all the external forces acting on the object must add up to zero. This is the equilibrium condition of concurrent forces, which are forces acting through a single point of the object. If the forces are not concurrent, there will be another equilibrium condition, which is not the topic of today.

Today, we will study the vector sum of three concurrent forces in static equilibrium. One of these forces, $\boldsymbol{W}$, is provided by a 2.00 N weight suspended from a vertical string. The other two forces, designated as $\boldsymbol{F}_{\mathbf{1}}$ and $\boldsymbol{F}_{\mathbf{2}}$, are applied by strings hanging from a backboard.

Vectors can be represented graphically by an arrow, or expressed numerically by its $x, y$, and $z$ components. In today's lab we consider two-dimensional vectors only (i.e. vectors with no $z$ component). In Problem 1, we will verify the static equilibrium condition by checking that the resultant force has zero components in both the $x$ and the $y$ directions. In Problem 2, static equilibrium is confirmed when the resultant force has zero length, or experimentally, when its length is shorter than the raw measurements' uncertainties. Since different methods are applied in each problem, you will write two separate reports for this lab.

## Problem 1

Purpose Verify the equilibrium condition of concurrent forces by the sum of their components.

Apparatus Draw a labelled diagram of the apparatus. (See Figure 1.) List any other equipment not labelled or illustrated, such as ruler, etc. If an identifying number is present on anything, give it too.


Figure 1

## Data

Hang both spring scales on the backboard without any strings, and adjust the zero readings so they read zero. Then hang the 2.00 N weight ( $\boldsymbol{W}$ in Figure 1) with the strings, adjust the hanging point so that the string from spring scale \#1 is as horizontal as possible. Tape a blank page for your report to the backboard so that the knot is near the center of the page.

Each partner is to include such a page in his or her lab report, with a descriptive title. Make sure to write down in your data section the information for the $\boldsymbol{W}$ vector as well.

On your paper, indicate WITH PENCIL the following four dots: (a) the location of the knot; (b) the right end of string 1 ; (c) the left end of string 2; and (d) the bottom end of string 3. Remove the page from the backboard and use these dots to draw three dotted lines: these are the directions of the vectors $\boldsymbol{F}_{\mathbf{1}}, \boldsymbol{F}_{\mathbf{2}}$ and $\boldsymbol{W}$, as indicated in Figure 1. Record the measured values shown on the spring scales in your own copies of the tables below. These are the magnitudes of the forces.

| Spring Force $F_{1}(\mathrm{~N})$ |
| :--- |
| Uncertainty in reading |
| Zero Reading |


| Spring Force $F_{2}(\mathrm{~N})$ |  |
| :--- | :--- |
| Uncertainty in reading |  |
| Zero Reading |  |
| Reading |  |
| Corrected Reading |  |

Corrected reading is not needed if zero reading is zero.

## Calculations

Remove the page from the backboard. Now we want to draw the three force vectors (magnitude and direction) on the page. Their directions are already drawn (they are along each of the dotted lines, outward from the knot's dot). Table 1 gives us the magnitudes in newtons (N), but to draw the forces (i.e. to know how long each arrow is) we need a scale that will allow us to represent newtons as centimetres. We have to convert the magnitudes of the forces to the lengths. A good scale to use is $1 \mathrm{~cm} \rightarrow 0.2 \mathrm{~N}$. Calculate the length of the vectors in cm and draw the vectors with the right directions and lengths, with arrow heads at the ends.


Figure 2

Now we will find the horizontal and vertical components of all three forces. $\boldsymbol{F}_{1}$ and $\boldsymbol{W}$ has only one component each. To find the components of $\boldsymbol{F}_{2}$, draw a right-angle rectangle on your page, as shown in Figure 2. Measure the horizontal and vertical sides of the rectangle in cm , convert them into forces in Newtons, to get $F_{2 \text { Horiz }}$ and $F_{2}$ vert.

Calculate the sum of all the horizontal components of the three forces, $R_{\text {Horiz }}$.
Calculate the sum of all the vertical components of the three forces, $R_{\text {Vert }}$.

## Conclusions

State the horizontal and vertical component of the resultant force, $R_{\text {Horiz }}$ and $R_{\text {Vert }}$. State if they satisfy the equilibrium condition.

## Problem 2

Purpose Verify the equilibrium condition by the graphic sum of the forces.
Apparatus Draw a labelled diagram of the apparatus (see Figure 3). Also, list any equipment not labelled or illustrated, such as ruler, etc. If an identifying number is present, give it too.


Figure 3

## Data

Set up the apparatus as shown in Figure 3. Tape a new blank page to the board. Mark the directions of the three forces on the page. Record the magnitudes of the forces in data tables.

Each partner is to include such a page in his or her lab report, with a descriptive title.

## Calculations

Draw the three forces, $\boldsymbol{F}_{\mathbf{1}}, \boldsymbol{F}_{\mathbf{2}}$ and $\boldsymbol{W}$, on your sheet, using the same scale of $1 \mathrm{~cm} \rightarrow 0.2 \mathrm{~N}$. This time we are going to find the vector sum of these three forces graphically, with the head-to-tail method.

To add $\boldsymbol{F}_{1}, \boldsymbol{F}_{2}$ and $\boldsymbol{W}$, we first move $\boldsymbol{F}_{2}$ so that its tail is touching the head of $\boldsymbol{F}_{1}$, then move $\boldsymbol{W}$ so that its tail is touching the head of $\boldsymbol{F}_{2}$. With three vectors joining head-to tail, the vector going from the tail of the first vector $\boldsymbol{F}_{1}$ to the head of the last vector $\boldsymbol{W}$ is the resultant vector $\boldsymbol{R}=\left(\boldsymbol{F}_{1}+\boldsymbol{F}_{2}\right)+\boldsymbol{W}$. Label $\boldsymbol{R}$ clearly on your diagram. Remember that when moving a vector, you must keep both its magnitude and direction. NOTE: DO NOT USE THE METHOD OF COMPONENTS HERE.

## Conclusions

Write your conclusion in sentences and state the magnitude of the sum of all three forces, i.e. the magnitude of $\boldsymbol{R}$. State if they satisfy the equilibrium condition.

## Discussion

Explain why you stated that equilibrium was or was not satisfied in each Problem. Use your uncertainties to justify your answers.

