Name: $\qquad$
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Desk \# $\qquad$
Date: $\qquad$

## Graphing

## Problem 1: Creating Linear Graphs

Read the description of the experiment carefully. Decide which measured quantities should be plotted on the $x$ - and $y$-axes, and how you will find the desired quantity and its uncertainty from the graph.

Example: The acceleration due to gravity, $g$, can be found by measuring the period, $T$, of a pendulum for various lengths, $L$. The relationship is given by:

$$
T=2 \pi \sqrt{\frac{L}{g}}
$$

| Raw data: $\quad T$ and $L$ | Desired quantity: $g$ |
| :--- | :--- |
| Graphing data: <br> $y$-axis: $T, x$-axis: $\sqrt{L}$ | Find the desired quantity from the slope of the graph: <br> slope $=\frac{2 \pi}{\sqrt{g}}$, so $g=\left(\frac{2 \pi}{\text { slope }}\right)^{2}$ |
| Uncertainty for graphing data: <br> $\delta y=\delta T, \delta x=\frac{\delta L}{2 \sqrt{L}}$ | Find the uncertainty of the desired quantity: <br> $\frac{\delta g}{g}=2 \frac{\delta \text { slope }}{\text { slope }}$ |

Calculation of $\delta x: \frac{\delta x}{x}=\frac{\delta L}{2 L}$, therefore $\delta x=\frac{\delta L}{2 L} x=\frac{\delta L}{2 L} \sqrt{L}=\frac{\delta L}{2 \sqrt{L}}$

1. The mass, $m$, of a sphere depends on the density, $\rho$, of the material and the diameter, $d$. By measuring the mass and diameter of several spheres of the same material, the density of the material can be calculated.

$$
m=\frac{\rho \pi d^{3}}{6}
$$

| Raw data: | Desired quantity: |
| :--- | :--- |
| Graphing data: <br> $y$-axis: $\quad, \quad x$-axis: | Find the desired quantity from the slope of the graph: |
| Uncertainty for graphing data: <br> $\delta y=$$\quad \delta x=$ | Find the uncertainty of the desired quantity: |

You may use the blank area below for scratch work:
2. The intensity $I$ of a point source of light depends on the distance, $r$, from the light source. By measuring the intensity at various distances, the power output, $P$, of the light source can be found.

$$
I=\frac{P}{4 \pi r^{2}}
$$

| Raw data: | Desired quantity: |
| :--- | :--- |
| Graphing data: <br> $y$-axis: $\quad, \quad x$-axis: | Find the desired quantity from the slope of the graph: |
| Uncertainty for graphing data: |  |
| $\delta y=\quad, \delta x=$ | Find the uncertainty of the desired quantity: |

3a. The period, $T$, of a mass, $m$, oscillating on a spring, depends on the "stiffness" of the spring, which is given by the spring constant, $k$. By measuring the period for different masses, the spring constant can be found.

$$
T=2 \pi \sqrt{\frac{m}{k}}
$$

Answer this question like the example on page 1:

| Raw data: | Desired quantity: |
| :--- | :--- |
| Graphing data: <br> $y$-axis: $\quad, \quad x$-axis: | Find the desired quantity from the slope of the graph: |
| Uncertainty for graphing data: |  |
| $\delta y=\quad, \delta x=$ | Find the uncertainty of the desired quantity: |

The choice of how to graph a relationship is not unique. Let's try Problem 3 again:
3b. Same as Question 3a, but we now will rearrange the equation as shown below. Let's plot $T^{2}$ on the $y$-axis and $m$ on the $x$-axis. This will lead to a different graph, but it is a linear graph too, and its slope also leads to the desired quantity. Finish this question.

$$
T^{2}=4 \pi^{2} \frac{m}{k}
$$

| Raw data: | Desired quantity: |
| :---: | :---: |
| Graphing data: <br> $y$-axis: $\quad, x$-axis: | Find the desired quantity from the slope of the graph: |
| Uncertainty for graphing data: $\delta y=\quad, \delta x=$ | Find the uncertainty of the desired quantity: |

4. For a given length of string, $L \pm \delta L$, a series of harmonic modes of frequency $f_{n}$ can be set up. By measuring the frequencies of the different harmonic modes of order $n$ on the string, the velocity of the wave, $v$, can be found. Note that $L$ has an uncertainty associated with its measurement.

$$
f_{n}=\frac{n v}{2 L}
$$

| Raw data: <br> $n(n=1,2,3 \ldots)$ and $f_{n}$ | Desired quantity: |
| :--- | :--- |
| Graphing data: <br> $y$-axis: $\quad, \quad x$-axis: | Find the desired quantity from the slope of the graph: |
| Uncertainty for graphing data: <br> $\delta y=$$\quad \delta x=$ | Find the uncertainty of the desired quantity: |

## Problem 2: Graphing Practice

Water balloons are dropped from a window on each floor of a 5 -story building. Since the balloons start from rest $\left(v_{0}=0\right)$, we can relate the height of the windows, $h$, to the drop time, $t$ and the gravitational acceleration $g$, with the equation:

$$
h=\frac{1}{2} g t^{2}
$$

The height of each window and the time for the balloon to reach the ground are recorded in the following data table. We will analyze this data using a linear graph to find an experimental value of $g$.

| Floor Number | Window Height $h(\mathrm{~m})$ | Time $t(\mathrm{~s})$ |
| :---: | :---: | :---: |
| Uncertainty | $\pm 0.2$ | $\pm 0.05$ |
| 1 | 2.5 | 0.69 |
| 2 | 6.0 | 1.08 |
| 3 | 9.5 | 1.37 |
| 4 | 13.0 | 1.61 |
| 5 | 16.5 | 1.83 |

First, as you did in Problem 1, plan to find $g$ and its uncertainty by graphing a linear relationship:

| Raw data: | Desired quantity: |
| :--- | :--- |
| Graphing data: <br> $y$-axis: $\quad, \quad x$-axis: | Find the desired quantity from the slope of the graph: |
| Uncertainty for graphing data: $\quad, \delta x=$ |  |
| $\delta y=$ | Find the uncertainty of the desired quantity: |

Next, unless you are graphing the raw data, you must prepare a "graphing data table". For each raw data point, use the equations above to calculate the graphing data and their uncertainties, $x, \delta x, y$, and $\delta y$, and record the results in the table below. Keep 4 significant digits when rounding. Put the units in the brackets. Unless there is already a linear relationship between the raw data, you must prepare such a graphing data table for every graph you create in the future.

| Floor Number | $x=\quad(\quad)$ | $\delta x=\quad(\quad)$ | $y=\quad(\quad)$ | $\delta y=\quad(\quad)$ |
| :---: | :--- | :--- | :--- | :--- | :--- |
| 1 |  |  |  |  |
| 2 |  |  |  |  |
| 3 |  |  |  |  |
| 4 |  |  |  |  |
| 5 |  |  |  |  |

Plot the graphing data on a $\mathrm{mm} / \mathrm{cm}$ graph paper. Graph paper with bigger divisions will not be accepted. After you locate the data points, draw their uncertainties with error bars. Then draw your "best fit" and "worst fit" lines. The "best fit" line should pass as close as possible to as many of the data points as possible. (It does not necessarily have to go through all of the points, nor the origin!) The "worst fit" line is the line that has a slope that is as different as possible from the "best fit" line, but still touches all of the boxes formed by the error bars.

On the graph, calculate the following:

- The slope of your "best fit" line: slope ${ }_{\text {best }}$ (it is also called "the slope")
- The slope of your "worst fit" line: slope worst
- The uncertainty of the slope: $\delta$ slope $=\mid$ slope $_{\text {best }}-$ slope $_{\text {worst }} \mid$


## Calculations

Calculate the gravitational acceleration $g$ from the slope:

## Uncertainty Analysis

Calculate the relative (also the percentage) uncertainty and the absolute uncertainty of the gravitational acceleration $g$ from the uncertainty of the slope:

## Conclusions

A reminder of the correct format of the conclusions: the absolute uncertainty must be rounded to 1 or 2 non-zero digits, and the value must stop at the same decimal place as the uncertainty.

The gravitational acceleration $g$ measured by dropping water balloons is
$\qquad$ . The reference value for $g$ in Vancouver is $(9.81 \pm 0.01) \mathrm{m} / \mathrm{s}^{2}$.

The experimental and reference values $\qquad$ (agree/do not agree) with each other, because $\qquad$
$\qquad$
$\qquad$ .

Remember to give the numerical values of the ranges, or the numbers showing the comparison between the absolute discrepancy and the total uncertainty. Since we did not carry out this experiment, we will not discuss the physical factors that are ignored.

