Name: _____

Continuous Charge Distributions – Prelab

Finish the exercises, and bring this handout to the lab

What is the problem?

We have learned to find the electric field of a point charge from Coulomb's Law:

$$\boldsymbol{E} = \frac{kq}{r^2}\hat{\boldsymbol{r}} \tag{1}$$

If there are many point charges, the field can be found by summing the fields due to each point charge:

$$\boldsymbol{E} = \sum_i \boldsymbol{E}_i, \text{ where } \boldsymbol{E}_i = \frac{kq_i}{r_i^2}\hat{\boldsymbol{r}}_i \tag{2}$$

If all the charges and their positions are known, we can always work out the above sum, although the calculations may be lengthy.

For a *continuous* charge distribution, the above sum becomes an integral over the volume of the charged body *V*:

$$\boldsymbol{E} = \int d\boldsymbol{E} = \int_V \frac{kdq}{r^2}\hat{\boldsymbol{r}} \tag{3}$$

For uniform charge distribution and simple geometry, the above integral may be worked out analytically (see Appendix A and B). But generally speaking, it is hard or even impossible to analytically work out the integral. No problem. With the "numerical integration" method that we introduce below, and the power of a spreadsheet program, we can find the electric field anywhere, of any charge distribution.

The "numerical integration" method

The idea of numerical integration is simple: we slice a continuous charge distribution into little pieces and view each piece as a point charge, then calculate the electric field using Eq. (2) rather than Eq. (3). In other words, we take summation rather than take integral. As long as the charge distribution is known, no matter it is non-uniform or irregular, summation can always be done.

While this process involves approximation, the result can reach high accuracy if the pieces are small. Needless to say, the price to pay for higher accuracy is the huge amount of calculations. Thanks to today's computer applications (for example, Microsoft's Excel), the extensive yet repetitive computations can be done without too much effort.

We will use this method to solve two simple questions. In this prelab, we use calculator, pen and paper; then we will come to the lab and use MS Excel to solve the same problems to a higher accuracy.

The steps are:

1. Draw the source charge and the field point;
2. Based on symmetry, decide which field component is zero, and which component needs to be calculated.
3. Cut the source of the field into small pieces (dices);
4. Calculate the field due to each piece, assuming they are point charges located at their centres;
5. Sum (by components) the fields of all the pieces.

Please note that you have to find the field components one by one, because the electric field is a vector. Do you remember how to add vectors? Adding vectors is harder than adding scalars, because you cannot add the magnitudes directly. Instead, you have to add vectors graphically or by components. Here, we need to add many vectors (one for each piece) and we will use the component method.

Problem 1 Field of a uniformly charged rod on a perpendicular bisector

The rod is 1.0 m long, and the total charge on it is $Q = 4$ nC. The field point P is at $y_P = 0.25$ m away from the centre.

Step 1: Sketch the source charge (the rod) and the field point P (Fig. 1).

Step 2: By symmetry, we know that E_x and E_z are zero, so we only need to calculate E_y.

Step 3: Slice the rod into 5 equal pieces. Choose a representative piece (preferably not an end or centre piece) and draw its electric field at P, as shown by ΔE.
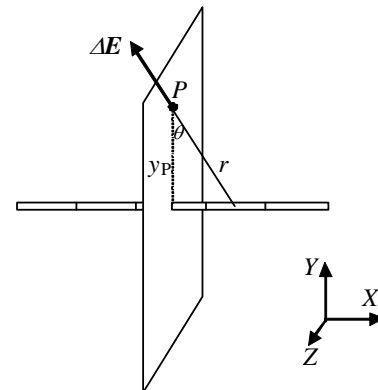

Fig. 1

Step 4: Calculate the field ΔE_y due to each piece using Eq. (2), Coulomb's Law. You need to know the Coulomb constant k, the charge of the piece Δq, the distance r and the angle between ΔE and y-axis θ.

$k = 8.99 \times 10^9$ Nm^2/C^2, $\Delta q = $ total charge Q divided by the number of pieces, $r = \sqrt{x^2 + y_P^2}$ where x is at the piece's centre and $y_p = 0.25$ m, and $\cos\theta = y_p/r$. The calculations for the first piece are done and you are required to finish the rest and fill the table below:

Piece #	Charge on the piece Δq (nC)	The piece's centre is at x (m)	Distance r (m) $r = \left(x^2 + y_P^2\right)^{1/2}$	$\cos\theta = y_p/r$	Field (y-component) (N/C) $\Delta E_y = \dfrac{k\Delta q}{r^2}\cos\theta$
1	0.8	−0.4	0.471699	0.529999	17.1315
2					
3					
4					
5					

Step 5: Sum ΔE_y of all 5 pieces. The result is $E_y = $ _____. (Answer: 258.9156 N/C)

These calculations can be simplified. Let Δx be the length of each piece ($\Delta x = 0.20$ m for 5 pieces), then $\Delta q = \dfrac{Q}{L}\Delta x$. We can rewrite the field to be $\Delta E_y = \dfrac{k\Delta q}{r^2}\cos\theta = \dfrac{k}{r^2}\left(\dfrac{Q}{L}\Delta x\right)\left(\dfrac{y_P}{r}\right) = \dfrac{kQy_P}{L}\dfrac{\Delta x}{r^3}$ where kQy_P/L is the same for all pieces. (It is 8.99 Nm/C if we substitute in the numbers.)

Piece #	Position of the centre of the piece x (m)	$\dfrac{\Delta x}{r^3} = \dfrac{\Delta x}{\left(x^2 + y_P^2\right)^{3/2}}$
1	−0.4	1.905614
2		
3		
4		
5		

Sum all $\Delta x/r^3$ and multiply by 8.99 Nm/C, you should again get $E_y =$ _____.

You can compare the numerical integration result with the analytical result in Appendix A.

(end of this page)

Problem 2 Field of a uniformly charged rod on axis

A uniformly charged thin glass rod of length $L= 1$ m and total charge $Q = 5$ nC is lying on the x axis with the left end at the origin. What is the electric field at a location $x_P = 1.5$ m?

Step 1: Sketch the source charge and the field point below.

Step 2: By symmetry, the only non-zero component is the _____ component.

Step 3: Slice the rod into 5 equal pieces and show that in your sketch. Also draw the field ΔE of a representative piece.

Step 4: Calculate the field due to each piece. To do that, first express ΔE_x in terms of k, Q, L, x_P and Δx, then factor out $kQ/L=44.95$ Nm/C. Express and calculate what is left for each piece and fill the table below. This second problem is simpler, because the field ΔE is pointing in the x-direction, so $\cos\theta$ is not needed.
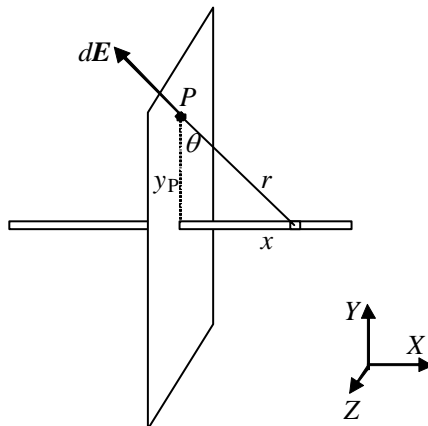
$$\Delta E_x =$$

Piece #	Position of the centre of the piece x (m)	
1		
2		
3		
4		
5		

Step 5: The sum of the last column is _____. Multiplying by kQ/L, we get $E_x =$ _____.

Appendix A Analytical Solutions

Problem 1: Calculate the field on the perpendicular bisector through the center of a uniformly charged thin rod.

By symmetry, $\mathbf{E} = \int d\mathbf{E}$ will be 0 in x and z directions. The only non-zero component of $\mathbf{E}$ is E_y:

$$E_y = \int dE_y = \int dE \cos\theta = \int dE \frac{y_P}{r} = \int \frac{kdq}{r^2} \frac{y_P}{r}$$

If λ is the "linear charge density" $\dfrac{Q}{L}$, then $dq = \lambda dx = \dfrac{Q}{L}dx$, and $r = \left(x^2 + y_P^2\right)^{1/2}$, therefore,

$$E_y = \int_{-L/2}^{L/2} \frac{k(Q/L)y_P}{\left(x^2 + y_P^2\right)^{3/2}} dx$$

Because it is a *thin* rod, the volume integral becomes a one dimensional integral along x-axis. This integral can be analytically calculated to be:

$$E_y = \frac{kQ}{y_P} \frac{1}{\sqrt{\left(\dfrac{L}{2}\right)^2 + y_P^2}}$$

If the length of the rod $L=1.0$ m, the total charge on it $Q = 4$ nC, and the field point P is at $y_P = 0.25$ m away from the centre, we can calculate the field to be _____.

Now solve Problem 2 analytically yourself. Please note that we have chosen two simple problems that can be solved analytically, which is not always the case.

Problem 2: Calculate the field of a uniformly charged thin rod of length L and total charge Q on its axis. The rod is lying on x-axis with the left end at the origin. What is the electric field at $x_P > L$?

First, sketch the rod and the field.

Then find an analytical expression of E_x at point P in terms of k, Q, L, and x_P.

Finally, substitute in the numbers ($k = 8.99 \times 10^9$ Nm^2/C^2, $L = 1$ m, $Q = 5$ nC, $x_P = 1.5$ m) to get the numerical result. You can use this exact result to compare with the result on page 4, and later with the result you get in the lab.

Appendix B Spreadsheet Basics

If you are familiar with Excel, skip this page; if you find this page is too hard to understand, ask your instructor to get a handout of "30-minute Excel Workshop".

Spreadsheets are useful for performing repetitive calculations, such as substituting different numbers into the same formula. Spreadsheets arrange numbers in a table, and each entry in the table is called a *cell.* The cells are referred to by their *addresses*: e.g., D5 refers to the cell in column D row 5.

Selecting cell(s)
Click a cell with the left button on the mouse to select it. To select more than one cell, click and hold the left button, and drag the pointer over the desired cells. To include more cells that are non-adjacent to your selection, hold the <Ctrl> key and select them. The selected cell(s) will be highlighted.

Changing entries
To change an entry in a cell, select it and its content will be shown in the "Formula Bar" near the top of the worksheet. Click it to edit the content. To finish editing, press the <Enter> key.

Typing a formula
You can enter a formula into a cell to perform a calculation. All formulas start with an equal sign "=". Suppose you want the cell C1 to be the sum of the numbers in A1 and B1. Type in cell C1 (omitting the quotation marks): "=A1+B1". You can also click on the cell A1 instead of typing "A1". After you finish typing, press the "Enter" key.

Copy the content to another cell
There are many ways to copy the content of one cell to other cells. The easiest is to select the cell(s) you want to copy, click and hold the small black square in the lower-right corner of the selection (the mouse pointer becomes a black cross), and drag to the cells that you want to copy to.

Another way to copy is to select the cell(s) you want to copy, choose "Copy" from the "Edit" menu or press the "Copy" button located on the toolbar, select the cell(s) you want to copy to, and choose "Paste" from the "Edit" menu or press the "Paste" button located on the toolbar.

Copying is very useful for Excel calculations. Suppose you want the cell C1 to be the sum of A1 and B1, cell C2 to be the sum of A2 and B2, etc. After typing "=A1+B1" in C1, you can then copy the content of C1 to C2, C3 … etc, and you will have the similar formulas in each of those cells. For example, C5 would end up as "=A5+B5".

Relative versus Absolute addresses

cell	entry	What happens if you copy cell B3 to another cell
B3	=A1^2	Cell B3 will be the number in cell A1 squared. If you copy cell B3 to cell B4, the formula in B4 will be "=A2^2", because B4 relative to B3 is like A2 relative to A1. The address A1 is therefore called a *relative address*.
B3	=$A$1^2	Cell B3 is also the number in cell Al squared. If you copy cell B3 to cell B4, the value in cell B4 will be "=$A$1^2", unchanged. The address $A$1 is fixed and therefore is called an *absolute address*.

If a cell containing a formula will not be copied to anywhere, then it doesn't matter whether it uses relative or absolute addresses.