Measuring the Magnetic Field with a Current Balance

Purpose
To measure the magnetic field strength inside a solenoid with a current balance.

Introduction and Theory

A current balance is used to measure a magnetic field. On one end of the balance is a copper conductor segment that allows a relatively large current to flow. On the other end, small masses can be added. When the current segment is placed in a magnetic field $\boldsymbol{B}$, the magnetic force on the current segment will be balanced about the pivot point by the weight of the small masses.

Suppose the current segment has length L_b and the current that runs through is I_b. If the current is perpendicular to the magnetic field $\boldsymbol{B}$, the magnitude of the magnetic force, F, is given by

$$F = I_b L_b B$$

If X_1 is the perpendicular distance from the current segment to the pivot point, the torque about the pivot point due to the magnetic force is

$$\tau_B = FX_1 = I_b L_b B X_1$$

This torque will be balanced by the gravity force acting on small wire weights added to the other end of the current balance. If n wire weights are put on the other end at distance X_2 from the pivot point, the torque due to gravity is

$$\tau_g = nwX_2$$

To balance, the two torques must be equal in magnitude:

$$I_b L_b B X_1 = nwX_2$$

so the magnitude of the magnetic field B can be calculated:

$$B = \frac{nwX_2}{I_b L_b X_1} = \frac{wX_2}{L_b X_1} \cdot \left(\frac{n}{I_b}\right)$$

We can measure the number of wire weights to balance a certain current and obtain a measurement of B. Better method would be varying the current and measure the number of wire weights needed to balance each current, plot n vs. I_b, and obtain the value of B from the slope of the plot.

The magnetic field that we are going the measure is the magnetic field inside a solenoid with a current running through the solenoid. We are going to measure the field at the center of the solenoid, where the magnetic field is essentially uniform.

A reference value for the magnetic field can be obtained by the geometric properties of the solenoid and the current running through the solenoid I_s.

Apparatus Draw a labelled diagram of the apparatus. See Fig 1. Obtain wire leads and a power supply from the shelves on the North wall.

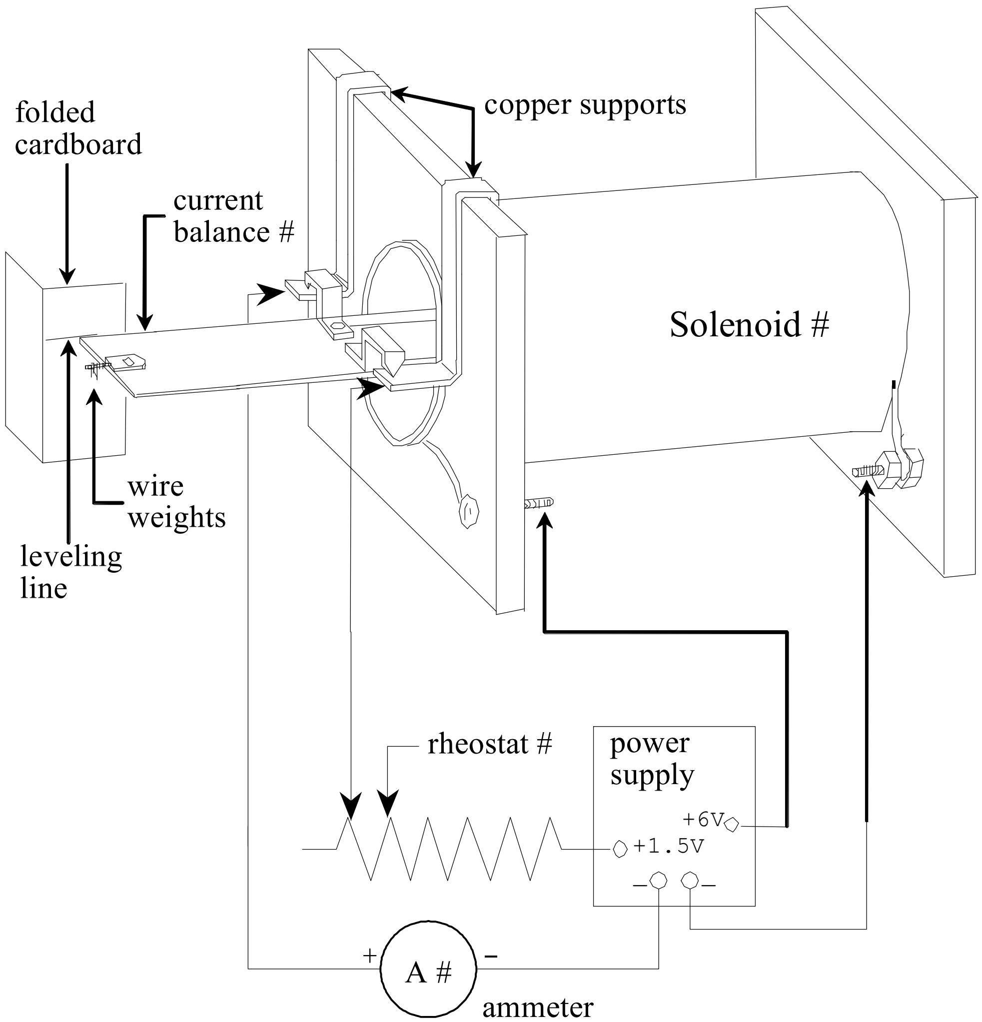

Fig 1

The following symbols are to be used:

B = the magnitude of the magnetic field inside the solenoid. This is the quantity to be measured;
I_b = the current in the current balance;
L_b = the length of the current segment which is perpendicular to the magnetic field;
w = the weight of each wire used to balance the magnetic force = $(7.98 \pm 0.05) \times 10^{-5}$ N;
n = the number of wire weights hanging from the screw;
X_1 = the moment arm for the magnetic force. See Fig 2;
X_2 = the moment arm for the wire weights. See Fig 2.

I_s = the current through the solenoid. It will change because the current will heat up the solenoid and change its resistance. Approximately, $I_s = (I_c + I_h)/2$, where I_c and I_h are the current through the solenoid before and after the solenoid is heated.

Data

1. Measure the dimensions of the current balance, obtaining the moment arms X_1 and X_2 and the length of the current segment L_b. X_2 should be measured up to where you are going to add the wire weights. L_b should be the average of the outer edge L_o and inner edge L_i. See Fig 2. Record your results in table format, paying attention to the uncertainties.

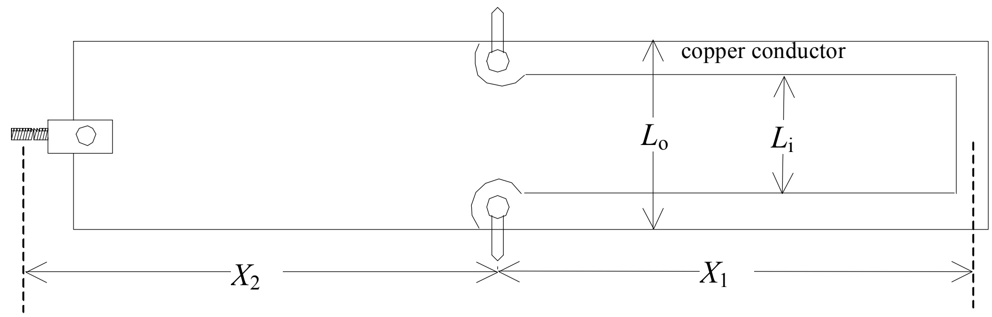

Fig 2

2. Set up a circuit to run 6V DC power across the solenoid. Measure I_c, the current through the solenoid when it is still cold, with the multimeter set to the 10A DC scale. Turn the power off after the measurement, to avoid unnecessary heating of the solenoid.

3. Set up the other circuit that runs 1.5V DC across the current balance, as shown in Fig 1. Since we need to change and measure the current of this circuit, we need to connect the rheostat and the multimeter. Make sure that your rheostat can modify the current and leave the knob at the half-way mark.

4. With power off and no wire weights, adjust the position of the pivot points on the current balance so that it moves freely without touching the inside of the solenoid. The current balance should end at a level position. Draw a line on the piece of folded cardboard to indicate this position.

5. Turn on the power to both circuits, noting as you do so, which way the outer (screw) end of the current balance moves. If the outer end moves up when the current is turned on, everything is as it should be. If it moves down, reverse the current through the balance. Now turn the rheostat knob to minimum current (maximum resistance).

6. Place 2 wire weights on the screw at the outer end of the current balance. Bring up the current through the balance until it is once again levelled at the mark on the cardboard. Record the current through the balance, I_b. Turn off both circuits to allow the solenoid to cool.

7. Repeat step (6) with 3, 4, 5, 6 wire weights on the screw and measure the current through the balance that is needed to level the balance each time. If you need more current, increase the power supply setting from 1.5V to 3.0V or 4.5V and turn the rheostat to a position of maximum resistance/minimum current. You should not go to the 6.0 V setting, as this will overheat the current balance circuit!!

8. Connect the ammeter to the solenoid and measure the current through the heated solenoid, I_h.

9. The major data taking is done. In order to get a reference value for the magnetic field inside the solenoid, we also measure the geometry of the solenoid. See Fig 3. The data we need are:
L_s: the length of the solenoid;
N: the number of turns of wire along the top layer of the solenoid;
D_0: outer diameter of the coil (measure any diameter three times at different places);
D_i: inner diameter of the coil (make sure not to include the thickness of the plastic tube).

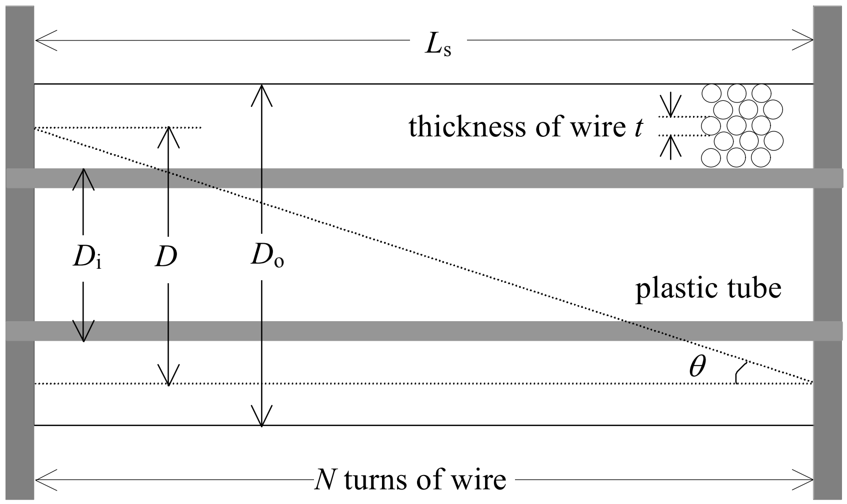

Fig 3

Calculations, Uncertainty Analysis and Conclusions

Plot the number of wire weights, n, vs. the current balance current, I_b, together with their uncertainties (if any). Find the slope of the best fit line. Write explicitly the relation between B and the slope. Calculate B and its uncertainties, based on the slope and the uncertainty in slope.

Calculate the reference value for the magnetic field B_{ref} based on the geometrical properties of the solenoid and the current through the solenoid:

$$B_{ref} = \frac{\mu_o N\, l\, I_s \cos(\theta)}{L_s}$$

where

μ_0:	the permeability of the free space	$4\pi\times10^{-7}$ Tm/A
N:	the number of turns of wire in one layer	
l:	the number of layers in the solenoid (this should be an *odd integer*)	given by D_0, D_i and the thickness of wire, $t = (0.14 \pm 0.01)$cm.
I_s:	the current through the solenoid	$I_s = (I_c + I_h)/2$
θ:	the angle shown in Fig 3	given by the average diameter D and L_s
L_s:	the length of the solenoid	

Calculate l, I_s and θ first, then calculate B_{ref}. Assume an uncertainty of $\pm\, 0.1 \times 10^{-2}$ T in B_{ref}.

Summarize the results in the Conclusion section. Compare the measured value and the reference value and discuss the agreement.

Disconnect all circuits and return the wire leads and the power supply.