

e/m : Charge to Mass Ratio of the Electron

This lab was adapted from Kwantlen University College's Determination of e/m lab.

Purpose

To determine the charge-to-mass ratio of the electron, e/m , using Helmholtz coils.

Introduction and Theory

Around the turn of the century, two dramatic experiments established the existence of the electron, a charged particle that is very much smaller than an atom in size and in mass. In 1897, J. J. Thomson analyzed the motion of cathode rays (electrons) as they passed through electric and magnetic fields. Thomson's investigations gave the following results:

- 1) The rays consist of particles that have a negative electric charge and a definite mass.
- 2) The charge-to-mass ratio of the electron was very large, about 2000 times that of a hydrogen ion, the lightest known ion.

From the second result, one can conclude that either the charge on the electron is very much bigger or its mass very much smaller than the hydrogen ion. Thomson believed that the latter was the case. Sometime later (around 1909), R.A. Millikan measured the charge on the electron and thereby confirmed Thomson's suspicions.

In this experiment, electrons are “boiled off” a heated cathode inside a specially designed tube (filled with helium at a pressure of about 10^{-2} mmHg), and then accelerated by a high potential difference between the cathode and anode. The kinetic energy gained by the electrons as they reach the anode is equal to the work done on them by the electric field, i.e.,

$$\frac{1}{2}mv^2 = qV \quad (1)$$

where m is the mass of the electron, v is the velocity of the electrons, q is the charge of an electron and V is the voltage between the electrodes that is used to accelerate the electrons.

The electrons emerge from a small aperture in the anode and enter a homogeneous (uniform) magnetic field produced by a pair of Helmholtz coils (described below). The force \mathbf{F} that the magnetic field exerts on the electrons is given by

$$\mathbf{F} = q\mathbf{v} \times \mathbf{B} \quad (2)$$

where \mathbf{B} is the magnetic field and \mathbf{v} is the velocity of the electrons.

One should note that the magnetic force \mathbf{F} is the vector product (also called cross product) of \mathbf{v} and \mathbf{B} . Such a force leads to very distinguishable motion patterns in a uniform magnetic field:

- (a) If \mathbf{v} is parallel to \mathbf{B} , then \mathbf{F} is zero. The electrons will move with constant velocity.

(b) If \mathbf{v} is perpendicular to \mathbf{B} , then \mathbf{F} is perpendicular to the velocity. The velocity will change in direction but not in magnitude. The electrons will undergo a uniform circular motion with radius that satisfies:

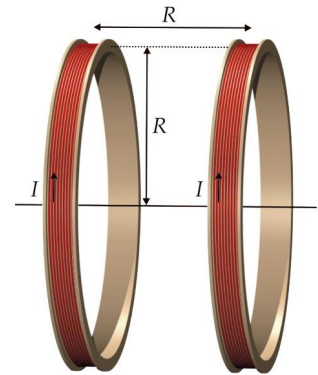
$$qvB = \frac{mv^2}{r} \quad (3)$$

Generally, the electron velocity has both parallel and perpendicular components and the motion is a spiral. In this lab, we let the electrons enter the magnetic field with velocity perpendicular to the magnetic field, so the electrons will move in a circle as in situation (b).

Combining Equations (1) and (3), noting the charge $q = e$, we obtain an expression for the electron charge-to-mass ratio e/m :

$$\frac{e}{m} = \frac{2V}{B^2 r^2} \quad (4)$$

All quantities except the magnetic field strength B in Equation (4) are directly measurable. We use Helmholtz coils to create a uniform magnetic field that can be easily calculated. The Helmholtz coils is a pair of conducting circular coils each having N turns, each carrying a current I , separated by a distance equivalent to the radius of the coils R (see the figure on the right).



The on-axis magnetic field due to a single wire loop of radius R carrying a current I is

$$B = \frac{\mu_0 IR^2}{2(R^2 + X^2)^{3/2}}$$

where μ_0 is the permeability of free space ($\mu_0 = 4\pi \times 10^{-7}$ Tm/A) and X is the distance to the center of the loop.

Let's say there are N current loops in each coil ($2N$ loops in total) and the currents are all flowing in the same direction (so their \mathbf{B} fields are all in the same direction), the total magnetic field is then

$$B = \frac{\mu_0 NIR^2}{(R^2 + X^2)^{3/2}} \quad (5)$$

At the centre of the axis of the Helmholtz coils, $X = R/2$, and Equation (5) simplifies to

$$B = \left(\frac{8}{\sqrt{125}} \right) \left(\frac{\mu_0 NI}{R} \right) \quad (6)$$

It can be proved that the magnetic field over a fairly large region near the centre of the Helmholtz coils is uniform and equal to the value given by Equation (6).

Prelab: Print this page and finish the questions before coming to the lab.

(1) The Helmholtz coil apparatus used in our lab has $N = 130$ and $R = 0.15$ m. Using equations given, show that:

$$\frac{e}{m} = \frac{2V}{(6.073 \times 10^{-7})I^2 r^2} \quad (7)$$

where the accelerating voltage V is in volts (V), the current in the Helmholtz coils I is in amperes (A) and the radius of the electrons beam r is in meters (m).

(2) Show that if r is held constant, the accelerating voltage will be proportional to the square of the current in the Helmholtz coils:

$$V = \left(\frac{1}{2} \frac{e}{m} (6.073 \times 10^{-7}) r^2 \right) I^2 \quad (8)$$

(3) Find the accepted value of e/m ($e = 1.602 \times 10^{-19}$ C and $m = 9.109 \times 10^{-31}$ kg).

Apparatus

The e/m experimental apparatus consists of a spherical tube (filled with helium at a pressure of about 10^{-2} mmHg) supported at the centre of a pair of Helmholtz coils. Within the spherical tube, electrons are emitted by a cathode which is indirectly heated by a filament. The cathode is connected to the negative terminal of the high voltage supply. The electrons are accelerated towards the anode attached to the positive terminal of the power supply. They pass through a hole in a grid which surrounds the cathode and another hole in the anode, and are thus formed into a beam. The path of the narrow beam produced by this electron “gun” is rendered visible by the glow discharge caused by ionizing collisions with helium gas inside the tube. To ensure a satisfactory circular beam, the electron path is perpendicular to the axis of the Helmholtz coils. There is a scale inside the tube for measuring the diameter of the path traced by the electron beam.

Setup

Caution: Do not leave the electron beam striking the surface of the tube for a prolonged period of time: it can take less than 3 minutes for the beam to bore a hole through and ruin the tube! Leave it deflected into a circular path or turn off the apparatus if you are not making any measurements. Also **do not leave the apparatus on at high voltages** for extended periods of time as it can “burn out.” Always turn off the power supply before connecting or disconnecting wires.

Connect as Figure 1:

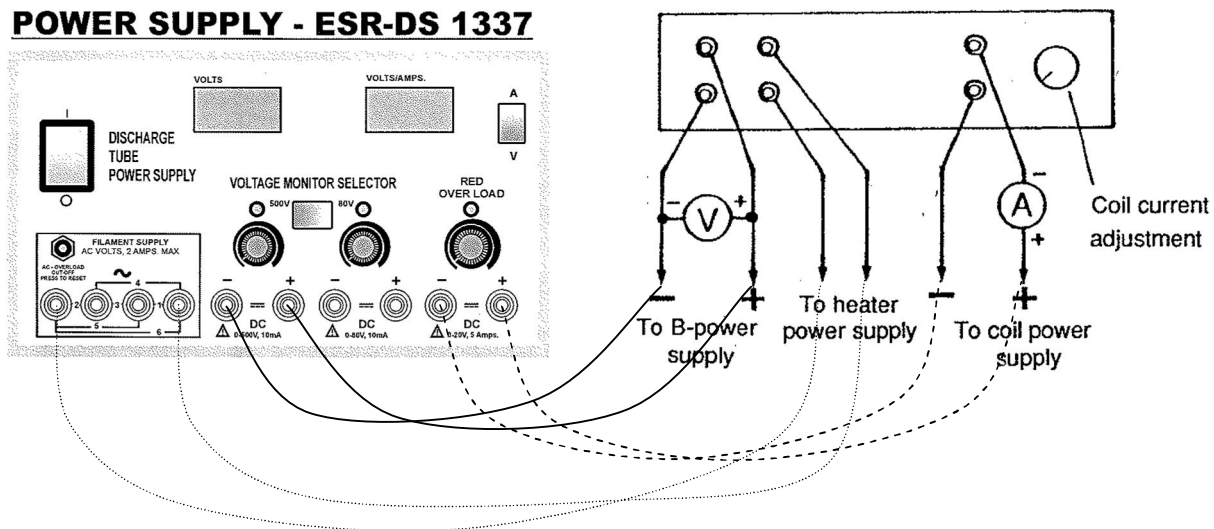


Figure 1

1. Connect the 6 V FILAMENT SUPPLY terminals on the power supply to the terminals marked HEATER on the e/m apparatus. The polarity does not matter.
2. Connect the 0—500V DC terminals on the power supply to the ANODE on the e/m apparatus, keeping positive to positive. Make sure the power supply’s 80 V – 500 V switch is set to the 500 V side, and that the voltage adjustment knob is turned fully counterclockwise (turned down to minimum).

3. Connect the 0—20V DC terminals on the power supply to the HELMHOLTZ COILS on the e/m apparatus, keeping positive to positive. The current through the coils can be adjusted both on the power supply and on the e/m apparatus. Turn the power supply knob fully clockwise (to maximum) and the control on the e/m apparatus to about half way.
4. Connect two multimeters to measure the voltage to the ANODE and the current through the Helmholtz coils, as shown in Figure 1.
5. Turn on the power supply and adjust the current through the coils immediately so that it is 2.0 A. **Do not exceed 2.0 A** since it is the maximum rating.

Data

1. Starting with an accelerating voltage of approximately 200 V, determine the current that produces a circular beam path with a large diameter (approximately 9-10 cm) so that its uncertainty is relatively low. (The scale visible in the tube measures the diameter in cm.) Record the diameter d and its uncertainty. Choose 6-8 approximately equal increments of voltage that cover a range of 200—500 V and for each V , adjust the current I until the beam path returns to the original diameter d , then record I and V from the multimeters. (They are more precise than the power supply display.) The accuracies of the multimeters are: V is $\pm (0.3\%+1)$ and A is $\pm (1.5\%+2)$. Ask instructor for the meanings of these accuracies.
2. Plot a graph to verify Equation (8) and determine e/m and its uncertainty from the graph.

This completes your data collection for writing the lab report. Following steps are optional:

3. Determine (a) the direction of the electrons' movement (clockwise or counter-clockwise) (b) the direction of the magnetic field (using the right hand rule) (c) the direction of the current in the coils (clockwise or counter-clockwise).
4. Predict how you will position a bar magnet near the tube to make the circle of the beam smaller. Carefully bring a bar magnet close to the tube to check your prediction. (Be extremely careful. We do not want magnets to break the tube!)
5. There are other ways to verify Equation (7) and measure e/m . (a) With the current through the coils fixed, measure how the radius of the electron beam changes with the accelerating voltage. (b) With the accelerating voltage fixed, measure how the radius of the electron beam changes with the current through the coils. Use spreadsheets to find e/m for each case, ignoring the uncertainties.

Lab Report

Your lab report should have following sections: Purpose, Apparatus, Data, Calculations, Uncertainty Analysis, Conclusion and Discussion. Insert your graph near the “Calculations” section, with slope and uncertainty of the slope calculated right on the graph. Calculate e/m in the “Calculations” section and its uncertainty in the “Uncertainty Analysis” section. State your final result in “Conclusion”. In the “Discussion” section, compare your result with the reference value you calculated in the Prelab. You can assume the reference value has no uncertainty. Discuss any physical factors that may affect your result. For example, does the earth field (0.05 mT, into the ground at an angle of about 70° to the horizontal) affect your result? The magnetic field we used is roughly 1.3 mT.