Measuring π by Areas

Alan Bird and David Cook
Department of Physics and Astronomy
Langara College
100 West 49th Ave, Vancouver, BC V5Y 2Z6, Canada

Begin with title, author(s) and the contact information

We measured the area of a circle and a rectangle by filling the areas with small steel spheres. Comparison of the sphere counts gives a measurement of the value of π. We obtained π = 3.12 ± 0.07 (± 2%), which agrees with the reference value.

Abstract: summarize what was done and what was found

Introduction

π, the ratio of a circle's circumference to its diameter, is an important constant that appears in many mathematical and physical formulae. Many people in history have tried to find its value; some had spent almost a lifetime[1]. Nowadays, computers can calculate π to practically an infinite precision (the current record is 5 trillion digits[2]), but finding π by oneself remains an interesting thing to do.

Reference to other people's work

Commonly, one finds π either from the circumference of a circle $c = \pi d$, or the area of a circle $A = \pi r^2$. Or, one can calculate π from a series like $\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \cdots$ [3]. We can also find π by physically measuring the circumference or the area of a circle. In this experiment, we will fill a circular box with small steel spheres, and the number of the spheres gives a measurement of the area of the circle and thus the value of π.

Method

We filled the bottom of a circular DVD box and a rectangular paper box with identical steel spheres, as seen in Figure 1.

Figure 1: A DVD box and a paper box filled with steel spheres

One picture is worth a thousand words!

The purpose of the rectangular box is to "calibrate" how much area each steel sphere takes. Therefore, rather than calculating the area of the circle directly from the number of the spheres, we will find the area of the circle by:

$$\frac{\text{area of circle}}{\text{area of box}} = \frac{\text{number of spheres in the circle}}{\text{number of spheres in the box}}$$

Before counting, we shook the boxes and adjusted the positions of the spheres by hand to maximize the number of spheres that can fill in. We also checked that all the spheres were touching the bottom.

Special precautions should be included in the Method part

Result

We counted 76 spheres in the DVD box and 149 spheres in the paper box. The numbers were each counted twice and two counts gave identical results. For reference, the spheres were (1.20 ± 0.05) cm in diameter.

We used a 30-cm student ruler to measure the lengths. To measure the diameter of the DVD box's bottom, we made 3 measurements at 3 different directions, from inner edge to inner edge. All three measurements gave same value of (12.40 ± 0.05) cm, therefore the diameter of the DVD box was (12.40 ± 0.05) cm.

To measure the length of the paper box, we made 4 measurements at the different places of the box. Unfortunately, because the box was not transparent, we had to put the ruler on top of the box and try to read the length of the bottom. The results we got were 21.3, 21.2, 21.3, and 21.2 in cm, with an average of 21.25 cm. Uncertainty was decided to be ± 0.1 cm.

Difficulties in the measurements should be described

The width was similarly measured to be 11.0, 11.1, 11.1 and 11.0 in cm, with an average of 11.05 cm. Uncertainty was also ± 0.1 cm.

This data combined gave π = 3.12 ± 0.07 (± 2%).

The detailed calculations and uncertainty analysis are not needed

Conclusion

By physically measuring the area of a circle, we found π to be 3.12 ± 0.07 (± 2%).

Discussion

Our result agrees well with the reference value 3.1415926(…)[2], but the uncertainty is big.

The numbers of the spheres were taken to have no uncertainty (see next paragraph), so the big uncertainty comes from the length measurements. If we can find a large, truly rectangular, rigid, transparent box, we can greatly improve the uncertainty. If we can achieve ± 0.05 cm in 50.00 cm, then we can bring down the uncertainty of π from 0.07 to 0.01.

Discuss the major uncertainties and systematic effects, and suggest improvements

We have ignored a number of systematic effects. Most obvious effect is that the spheres do not cover the entire area: there are gaps between them. In other words, our counted numbers do have uncertainty, and the "true number", the number of spheres covering the area without gaps, should be larger. Naturally, the smaller the box is, the more significant are the gaps, and more the counted number is off. Because our DVD box was smaller than the paper box, 76 differs more from the "true number" than 149, that is, the correct area ratio should be larger than 76/149. This could be the reason why our result 3.12 is smaller than the true value, although with our current uncertainty, the difference is insignificant. This systematic effect could be reduced by choosing smaller spheres or larger boxes, and by choosing similar sizes for both boxes.

Acknowledgement

We thank Mr. Eric Pie for helpful discussions.

Reference

1. Petr Beckmann, *A History of Pi* (St. Martin's Press, New York, NY, 1971.)
2. http://en.wikipedia.org/wiki/Pi
3. http://www.jimloy.com/geometry/pi.htm