PROBLEM OF THE MONTH

Should You Be So Inclined

June 2023 - Langara Physics & Astronomy

0 Introduction

0.1 THE PRIZE

This month you could win a **$20 Starbucks Gift Card**! In addition, correctly answering *any* of the 6 questions will automatically place you on our **Display Case Leaderboard**, whose semester-long Top Solver will win a **Grand Prize** at the end of the semester!

0.2 THE RULES

Each question you solve earns you "Entries" into a draw which will take place at the end of the month. The more entries you have, the better your odds- but it only takes one to have a shot at winning! Each question is worth a different number of entries roughly proportional to its difficulty. You **do not** need to answer all of the questions. You also do not need to answer previous questions in order to submit your solution to any subsequent question. Partial solutions will be considered!

0.3 SUBMISSIONS

Your solutions must be either: put in Hand-In Slot #10 outside room T340; handed directly to me (Alex); or emailed to achoinski@langara.ca, **before midnight on June 30th**. Questions about the contest or problems can be directed to achoinski@langara.ca.

0.4 INTRODUCTION TO THIS MONTH'S PROBLEMS

This month, we tackle questions of how objects slide down **inclined planes**. These problems haunt much of one's first years in physics. They are classic problems whose solutions are part of a good physicist's toolkit. Here you will find some introductory questions about free-fall and sliding down a ramp with and without friction, followed by some more novel plays on those classics.

Note: Assume all problems take place near to the surface of the Earth, such that we may assume a constant downward acceleration of $\mathbf{g}$.

Happy physics-ing, and *good luck!*

Online PDF Version Found Here!

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1 Dropping The Ball

Difficulty: ★☆☆☆☆
Worth: 1 Entry

Question: Write down the formula for the time $\mathbf{\Delta t}$ it would take a mass $\mathbf{m}$ to hit the ground if released from a height $\mathbf{h}$. Ignore air resistance

2 Ramping Things Up

Difficulty: ★☆☆☆☆
Worth: 1 Entry

Question: Write down the formula for the time $\mathbf{\Delta t}$ it would take a mass $\mathbf{m}$ to hit the ground while sliding down a ramp which falls a height of $\mathbf{h}$ over a length of $\mathbf{L}$ metres (see Fig. 1 below). You may present your answer in terms of $\mathbf{h}$ and $\mathbf{L}$ *or* in terms of θ. Ignore friction.

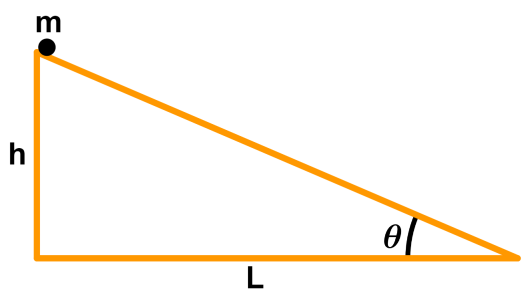

Figure 1: A Simple Ramp

3 Truth, Or Friction?

Difficulty: ★★☆☆☆
Worth: 2 Entries

Question: Given the same setup as **Question 2** (see Fig. 1), now consider that there *is* a coefficient of kinetic friction $\mu_\mathbf{k}$. Take the coefficient of static friction $\mu_\mathbf{s}$ to be zero (i.e. it begins sliding immediately). Find the new expression for $\mathbf{\Delta t}$, in terms of $\mathbf{m}$, $\mu_\mathbf{k}$, $\mathbf{h}$ and $\mathbf{L}$ *or* θ.

4 Double Trouble

Difficulty: ★★⯪☆☆
Worth: 2 Entries

Question: Write down the formula for the time $\mathbf{\Delta t}$ it would take a mass $\mathbf{m}$ to hit the ground while sliding down a two-section ramp, whose geometry is specified in Fig. 2 below. Present your answer in terms of $\mathbf{h}$ and $\mathbf{L}$, *not* θ. Ignore friction.

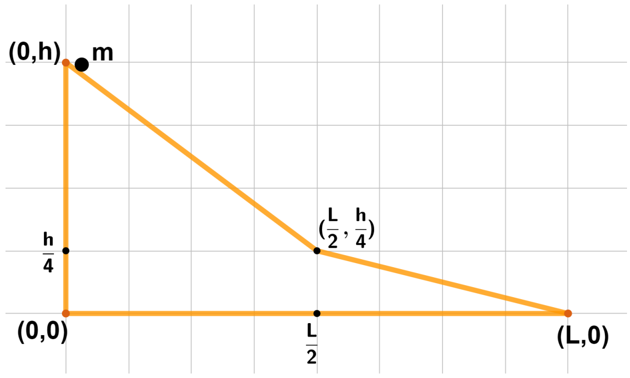

Figure 2: A two-section ramp.

5 A Mysterious Curve

Difficulty: ★☆☆☆☆ - ★★★★★, Up To You!
Worth: 1-3 Entries (see below)

Question: State the time $\mathbf{\Delta t}$ it would take a mass $\mathbf{m}$ to hit the ground while sliding down the curve as seen in Fig. 3 (assume all given dimensions are in metres). The formula of the curve is given. **Note:** This is an exercise in approximation - it cannot be solved exactly. You will receive **one entry** if you are within **10%** of the true time - you will receive **two entries** if you are within **1%** - and receive ***three entries*** if you are within **0.1%**. Make a guess! Take your (and its) time!

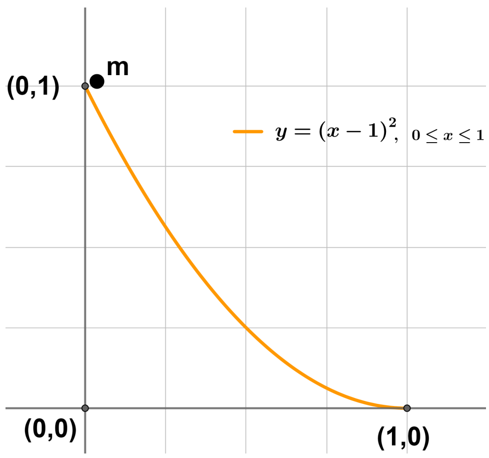

Figure 3: The non-linear ramp **Question 5** considers.

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6 An Arbitrarily Mysterious Curve

Difficulty: ★★★★★ + *Calculus*
Worth: 5 Entries

Question: Write down an expression giving the time $\mathbf{\Delta t}$ it would take a mass $\mathbf{m}$ to hit the ground (y = 0) while sliding down an arbitrary function:

$$f : [0, 1] \to [0, 1], \quad (1)$$

where f has the following properties:

- f(0) = 1

  i.e. the ramp has a starting height of 1

- f(1) = 0

  i.e. the ramp has a length of 1

- $\forall$x $\in$ (0,1], f(x)<1

  i.e. at no point in the run of the ramp (for no x) does f(x) re-attain its starting height of 1.

- f is twice differentiable on [0,1]

  i.e.: the second derivative of your function is also a well define function on the domain [0,1]. In practice this just means that if your expression includes a second derivative of f (wink), you're safe in knowing it's well defined.

Hint: Think conservationally.