Uncertainty

What it is, where we need it, how does uncertainty propagate

What is uncertainty?

- A police says your car was travelling at a speed of 70 km/h. What does he really mean? Now he has to give you a better answer. He says you were speeding because the speed limit is 50 km/h and

  the speed of your car was (70 ± 5) km/h

  - 70 km/h is the **value** of the speed, also called the best estimate: v;
  - 5 km/h is the **uncertainty** of the speed: δv;
- The **value** and the **uncertainty** together define a **range** (65 to 75 km/h) where the "true" speed most likely falls within. i.e.: If someone were taking the same measurement, he/she would likely get a value within this range.
- How likely? It depends. For example, 68% likelihood if using "standard deviation" as the uncertainty for a normal distribution. We will try "70% or more" for our labs.
- None of above terms is exact: they are all uncertain.

Absolute uncertainty and relative uncertainty

- Uncertainty that describe the range of the data is also called "the absolute uncertainty". It has the same units as the value. Symbol is δA.
- Is it a big uncertainty or a small uncertainty? 1 meter is a big uncertainty for the size of our lab, but is a small uncertainty for the distance to the moon. The precision of the measurement is reflected by the relative uncertainty $\delta A/A$.

Speed of the car v is (70 ± 5) km/h

$$\text{Relative uncertainty } \frac{\delta v}{v} = \frac{5\text{ km/h}}{70\text{ km/h}} = 0.0714 \approx 7\%$$

- Absolute uncertainty has units, but relative uncertainty has no units.
- Absolute and relative uncertainty: knowing one = knowing them both.

Where is uncertainty in our lab report?

1. Purpose: no
2. Apparatus: no
3. Data: Yes. All data must have uncertainties (except counting a small integer). Decide uncertainty based on (1) measuring devices (2) the situation.
4. Calculations: no
5. Uncertainty Analysis: Yes. Here we show how the uncertainties of the raw data propagate into the uncertainty of the result.
6. Conclusion: Yes. You must report result with uncertainty.
7. Discussions: Yes. Compare the result to a reference number to see if they agree within uncertainty and discuss the sources of uncertainty.

Uncertainty Propagation

No more sig. fig. rules!

Unc. of data → Unc. of result

You need

- The equation that calculates the result
- Propagation rules

Propagation Rule # 1: addition and subtraction

- If $z = x + y$ or $z = x - y$, then $\delta z = \delta x + \delta y$

Example:

Propagation Rule # 1: addition

- If $z = x + y$, then $\delta z = \delta x + \delta y$

$n_1 = 130 \pm 3, n_2 = 50 \pm 2, n_3 = 20 \pm 1$

Total number of the beads:
$n = n_1 + n_2 + n_3 = 130 + 50 + 20 = 200$

$\delta n = \delta n_1 + \delta n_2 + \delta n_3 = 3 + 2 + 1 = 6$

So the total number of beads is 200 ± 6.

Propagation Rule # 1: subtraction

- If $z = x - y$, then $\delta z = \delta x + \delta y$

$n = 200 \pm 2, n_{\text{out}} = 70 \pm 7$

Number of beads left in the bottle:
$n_{\text{in}} = n - n_{\text{out}} = 200 - 70 = 130$
$\delta n_{\text{in}} = \delta n + \delta n_{\text{out}} = 2 + 7 = 9$

So the number of beads left in the bottle is 130 ± 9.

Propagation Rule # 2: Multiplication and division

- If $z = xy$ or $z = x/y$, then $\frac{\delta z}{z} = \frac{\delta x}{x} + \frac{\delta y}{y}$

Example: Adam rides his bike to Langara. He covers (3.2±0.2) km in (10±1) minutes. What is his average speed?

$$v = \frac{d}{t} = \frac{3.2 \text{ km}}{10 \text{ min}} = 0.32 \text{ km/min}$$

$$\frac{\delta v}{v} = \frac{\delta d}{d} + \frac{\delta t}{t} = \frac{0.2}{3.2} + \frac{1}{10} = 0.1625$$

$$\delta v = \frac{\delta v}{v} \times v = 0.1625 \times 0.32 \text{ km/min} = 0.052 \text{ km/min} \approx 0.05 \text{ km/min}$$

So his average speed is (0.32 ± 0.05) km/min (± 16%).

Propagation Rule # 3: Multiplication by a constant

- If $z = kx$ then $\delta z = k\delta x$, or $\frac{\delta z}{z} = \frac{\delta x}{x}$

Example: Betty bought 10 bottles of beads online. Each bottle contains 200±2 beads, which is a 1% uncertainty. For 10 bottles, it will be 2000 ± 20 beads, which is also a 1% uncertainty. Multiplying by 10 increases the absolute uncertainty from 2 to 20, but does not change the relative uncertainty.

Use absolute uncertainty:

$$n = 10n_1 = 10 \times 200 = 2000$$
$$\delta n = 10\delta n_1 = 10 \times 2 = 20$$

Use relative uncertainty:

$$n = 10n_1 = 10 \times 200 = 2000$$

$$\frac{\delta n}{n} = \frac{\delta n_1}{n_1} = \frac{2}{200} = 0.01$$
$$\delta n = \left(\frac{\delta n}{n}\right) n = 0.01 \times 2000 = 20$$

Both methods get the same result: 10 bottle of beads will have 2000 ± 20 beads.

Propagation Rule # 4: Powers

- $If\ z = x^n\ \ then \frac{\delta z}{z} = |n| \frac{\delta x}{x}$ *Don't mix up exponent and constant!*

Example: Adam is painting a square wall. Its length and width are both (2.00±0.05) m.

$A = a^2 = (2.00\ m)^2 = 4.00\ m^2$

$\frac{\delta A}{A} = 2\frac{\delta a}{a} = 2 \times \frac{0.05}{2.00} = 0.05$

$\delta A = \frac{\delta A}{A} \times A = 0.05 \times 4.00\ m^2 = 0.2\ m^2$

The area of the wall is (4.0 ± 0.2) m^2 (± 5%).

Apply the rules to many variables

$$z = a + b - c + d \rightarrow \delta z = \delta a + \delta b + \delta c + \delta d$$

$$z = \frac{xy}{uvw} \rightarrow \frac{\delta z}{z} = \frac{\delta x}{x} + \frac{\delta y}{y} + \frac{\delta u}{u} + \frac{\delta v}{v} + \frac{\delta w}{w}$$

$$z = \frac{x^3 y}{4\sqrt{v}w} \rightarrow \frac{\delta z}{z} = 3\frac{\delta x}{x} + \frac{\delta y}{y} + \cancelto{0}{\frac{\delta 4}{4}} + \frac{1}{2}\frac{\delta v}{v} + \frac{\delta w}{w}$$