## Measurements in the Physics Lab: Advanced

For "Measurements: Basic", see
http://www.langaraphysics.com/measurementsbasic.pdf.

## Review of Taking Data

You should ensure the instruments are reading correctly. In particular, you should check zero readings before taking data.

When recording data, record all the displayed digits for digital devices, and record all accurate digits plus one estimated digit for analog devices.

Data must have

- Explanations: describe what the data represents in words.
- Units: use the unit of the measuring device.
- Uncertainties: describe the possible range of the measurement.

In order for people to understand, your data must have clear meanings and well organized.

## Review of Uncertainty

## Terminology

For a measurement of the length of a desktop: $L=(1.830 \pm 0.005) \mathrm{m}( \pm 0.3 \%)$

- 1.830 m is called the value, or the best estimate of the length.
- 0.005 m is called the absolute uncertainty, or simply the uncertainty, noted as $\delta L$.
- The absolute uncertainty divided by the value is called the relative uncertainty, noted as $\frac{\delta L}{L}$.
- $0.3 \%$ is called the percentage uncertainty. It is the relative uncertainty expressed in the percentage format, $\left(\frac{\delta L}{L} \times 100 \%\right)$.

The (absolute) uncertainty has the same units as the value. The relative uncertainty and the percentage uncertainty have no units as the units cancel during the division.

## How do you decide on the uncertainty of measured data?

The uncertainty should be the bigger of two factors: one from the measuring device and method, one from the fluctuation or the scatter of possible values.

- The uncertainty is as least as large as the precision of the measuring device: if it is not given, we take it to be $\pm 1$ in the last digit for digital devices, and half of the smallest division for analog devices. Nonetheless, the uncertainty is often larger because the uncertainty also comes from factors other than the measuring devices, such as the method or the object itself, which is situationdependent.
- If it is possible to repeat the measurement, then we can use the scatter of the measured values:

$$
\text { scatter }=\frac{\text { range }}{2}=\frac{\text { largest measurement }- \text { smallest measurement }}{2}
$$

If it is not possible to repeat the measurements, or repeating yields identical values, then you have to decide the uncertainty based on your feelings and experience. It is often helpful to ask yourself "what values are possible?"

## Uncertainty Propagation

## How do you determine the uncertainty of calculated results?

Just like data, the result also has uncertainty. In PHYS 1114 and PHYS 1118, we do not require you to find the uncertainty of the result explicitly. Starting now, you need to determine the uncertainty of the result.

If you have multiple values of the same result, you can assign the uncertainty to be the scatter of the values, using (maximum number - the minimum number)/2. More often though, you have only one result calculated from the data. In this case, the uncertainty of the result is determined by the uncertainty in the data and the equations used in the calculation.

Calculating the uncertainty of the result from the uncertainty of the data and the equations is called uncertainty propagation. Other names for this process are uncertainty analysis, precision analysis or error analysis. Note however: Uncertainty analysis is NOT an examination of possible mistakes. There are specific mathematical rules to follow in order to combine your uncertainties correctly.

For uncertainty propagation rules, see
http://www.langaraphysics.com/propagationrules.pdf.
For uncertainty propagation examples, see
http://www.langaraphysics.com/propagationexamples.pdf.
How do you report the result with uncertainty?
You report the result in the "Conclusion" section of the report. You must use complete sentences to fully answer the purpose of the lab. Any numerical result must be rounded properly, based on the uncertainty:

- Round uncertainties to 1 or 2 non-zero digits.
- Round the value to the same decimal precision as the uncertainty.

It should be easy to tell whether a result is in the correct format. For example, $(50.4 \pm 0.3) \mathrm{m} / \mathrm{s}$, $(0.40 \pm 0.02) \mathrm{kgm}^{2},(8.00 \pm 0.15) \mathrm{N}$ are all in the correct format, while $(50.37 \pm 0.3) \mathrm{g},(0.4 \pm 0.02) \mathrm{m}$ and $(83.154 \pm 2.415) \mathrm{m} / \mathrm{s}$ are wrong because the decimal places do not match or the uncertainty has too many digits.

How do you compare your result with expectation or reference value?
After you get the final result, you should compare it with some accepted value. Two values agree within uncertainty if the ranges that they cover overlap. It is easy to see what this means with a comparison graph below:

$x_{1}$ and $x_{2}$ agree within uncertainty since their ranges overlap, whereas $x_{1}$ and $x_{3}$ do not agree within uncertainty since their ranges do not overlap.

One can usually tell by inspection if the measurements agree. For example, $x_{2}$ can be as big as $128+8$ $=136 \mathrm{~km}$, and $x_{3}$ can be as small as $137-4=133 \mathrm{~km}$, so they overlap and agree.

If the agreement between two values is not obvious, you can clarify it by drawing a comparison graph like above, or by a simple calculation. In order for agreement, the difference between the two values must be less than the sum of both uncertainties:

- $x_{1}$ and $x_{2}$ agree since

$$
|128 \mathrm{~km}-115 \mathrm{~km}|<(8 \mathrm{~km}+7 \mathrm{~km})
$$

- $x_{1}$ and $x_{3}$ do not agree since

$$
|137 \mathrm{~km}-115 \mathrm{~km}|>(4 \mathrm{~km}+7 \mathrm{~km}) .
$$

A discussion of the agreement (or not) between a measured/calculated/experimentally determined result and a reference value from theory or a reference source should be the basis of a report's "Discussion" section. The statement of agreement (or not) should be clear and meaningful, thus displaying your understanding of the results of the experiment. For details about the discussion, see http://langaraphysics.com/110111251225report.pdf.

