Uncertainty Propagation Rules

Rule 1: Addition or Subtraction	
The *absolute uncertainty* in the result of an addition or a subtraction of variables is equal to the sum of the individual *absolute uncertainties*. For two variables, "Rule 1" can be expressed in two forms as below. The rule also applies to three or more variables.	
If $z = x + y$ or $z = x - y$, then $\delta z = \delta x + \delta y$	$\delta(x+y) = \delta x + \delta y$ and $\delta(x-y) = \delta x + \delta y$

Rule 2: Multiplication or Division	
The *relative uncertainty* in the result of a multiplication or a division of variables is equal to the sum of the individual *relative uncertainties*. For two variables, "Rule 2" can be expressed in two forms as below. The rule also applies to three or more variables.	
If $z = xy$ or $z = \frac{x}{y}$, then $\frac{\delta z}{\lvert z\rvert} = \frac{\delta x}{\lvert x\rvert} + \frac{\delta y}{\lvert y\rvert}$	$\frac{\delta(xy)}{\lvert xy\rvert} = \frac{\delta x}{\lvert x\rvert} + \frac{\delta y}{\lvert y\rvert}$ or $\delta(xy) = \left(\frac{\delta x}{\lvert x\rvert} + \frac{\delta y}{\lvert y\rvert}\right)\lvert xy\rvert$

Rule 3: Multiplication by a Constant	
If $z = kx$, then $\delta z = k\delta x$ or $\frac{\delta z}{\lvert z\rvert} = \frac{\delta x}{\lvert x\rvert}$	$\delta kx = k\delta x$
Examples: (1) $\delta(2A) = 2\delta A$. (2) $\frac{\delta(2A)}{2A} = \frac{\delta A}{A}$	

Rule 4: Powers	
If $z = x^n$, then $\frac{\delta z}{\lvert z\rvert} = \lvert n\rvert\frac{\delta x}{\lvert x\rvert}$	$\frac{\delta(x^n)}{x^n} = \lvert n\rvert\frac{\delta x}{x}$ or $\delta(x^n) = \lvert n\rvert x^{n-1}\delta x$
Examples: (1) $\frac{\delta(x^3)}{x^3} = 3\frac{\delta x}{x}$. (2) $\frac{\delta(\sqrt{x})}{\sqrt{x}} = \frac{1}{2}\frac{\delta x}{x}$	

Rule 5: Any Function of One Variable	
If $z = z(x)$, then $\delta z = \left\lvert\frac{dz}{dx}\right\rvert\delta x$	
Example: $\delta(x^3) = 3x^2\delta x$.	