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Desk # _______________
Date: _________________

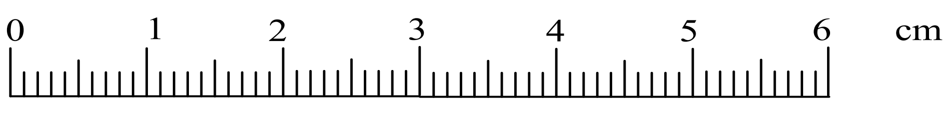
Phys 1101
Uncertainty Worksheet

Most of the concepts introduced in this lab are explained in more detail in the Langara Physics Department on-line lab manual, in the Introductory Materials section.
http://www.langaraphysics.com/measurementsbasic.pdf
http://www.langaraphysics.com/measurementsadvanced.pdf
http://www.langaraphysics.com/propagationrules.pdf
http://www.langaraphysics.com/propagationexamples.pdf

These documents are quite short and self-contained.

Single Measurement Uncertainty

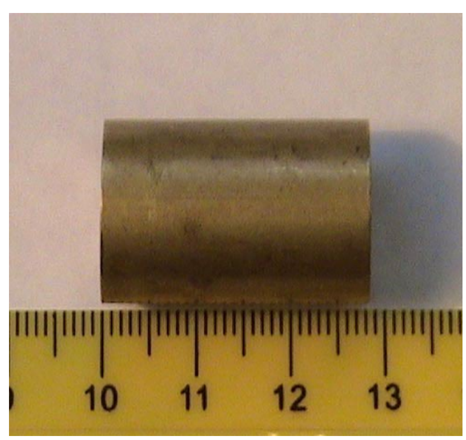
The uncertainty in a single measurement is influenced by many things, and is often an estimate based on common sense and experience. Operator skill with the equipment is probably the biggest source of uncertainty. The one who is doing the measuring is the best judge of this, not your instructors!



Read the length of the horizontal line in above diagram, using the given scale. Remember that the value should be kept to the same decimal places as the uncertainty.

The length of the line: l = (_______ ± _______) cm

Read, from the pictures below, the size of an apple, and the length of a well-machined cylinder. The units of the ruler is cm. Do NOT write on the pictures.



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The size of the apple is *D* = (_______ ± _______) cm.

The length of the cylinder is *L* = (_______ ± _______) cm.

Hint: do you think *D* and *L* above should have same uncertainty? Explain.

Scatter from Multiple Measurements

When we measure the same quantity many times, the results may vary or scatter, leading to a bigger uncertainty. Each individual measurement is valid, and the uncertainty in the average must reflect ALL possible valid values.

Measure the diameter of this personal size pizza. Notice that the pizza is not a perfect circle. Measure in 3 different directions and fill up the data table.

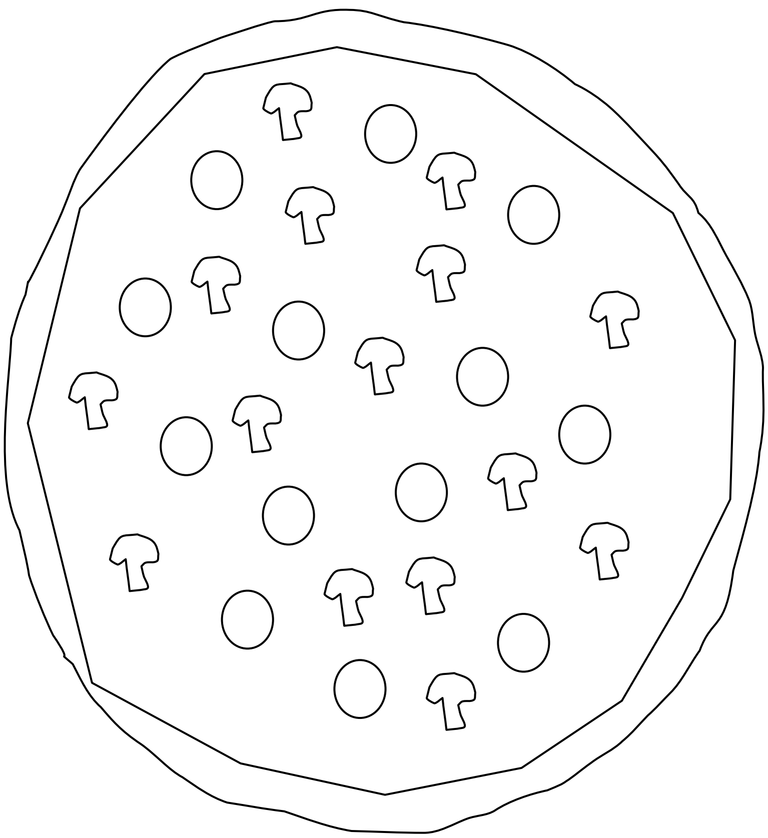

Table 1: Diameter of a Pizza *D* (cm)

Reading 1	
Reading 2	
Reading 3	
Average *D*	
Uncertainty	

Note 1: uncertainty above is calculated from scatter: (max−min)/2.
Note 2: 5 digits are kept for the average and the uncertainty for calculating purposes.

Here the ruler's precision (±0.05cm) and the uncertainty from each measurement (±0.1cm or 0.2cm) are much smaller than the scatter of the diameter values, so we will use the scatter to be the final uncertainty of the diameter of the pizza.

Uncertainty Propagation through equations: Review the propagation rules and examples at the back of this handout.

1. Write the expression for either the relative uncertainty or the absolute uncertainty.
(Should you start with absolute uncertainty *δq*, or relative uncertainty *δq/q*? For sum/difference, start with absolute uncertainty. For other types, find the relative uncertainty first.)

$F = ma$	$\left(\frac{\delta F}{F}\right) = \frac{\delta m}{m} + \frac{\delta a}{a} \Rightarrow \delta F = \left(\frac{\delta F}{F}\right) \cdot F$
$m = m_1 + m_2$	
$V = \frac{1}{6}\pi D^3$	
$q = \sqrt{xy}$	
$T = 2\pi\sqrt{\frac{l}{g}}$	
$L = \frac{1}{2}(L_1 + L_2)$	

2. Work out some numbers

Find the numerical value for q and its absolute and relative uncertainties. For products, find the value of the relative uncertainty first, and then multiply by q to get the absolute uncertainty. Quote your answer properly.

$A = (3.3 \pm 0.5)\ \mathrm{m}^2$, $B = (2.2 \pm 0.1)$ m, $C = (5.5 \pm 0.2)$ m.

$$q = \frac{3AB}{C}$$

Answer: $(4.0 \pm 0.9)\ \mathrm{m}^2$ (± 23%)

$q = 3AB^2/C$ Answer: $(9 \pm 2)\ \mathrm{m}^3$ (± 28%)

3. More complete and meaningful examples. Quote your answer in proper format, as in the examples.

a) To determine the amount of wallpaper q needed for a square room, a decorator measures:

Wall height,	$h = 2.49 \pm 0.01$ m
Wall width,	$w = 2.10 \pm 0.01$ m
Area of windows and doors,	$A = 3.51 \pm 0.06$ m^2

Find $q = 4hw - A$ and its uncertainty. (Answer: q = (17.4 ± 0.2) m^2 (± 1%))

b) What is the area (in m^2) of the pizza on page 2 of this lab?

Uncertainty Propagation Examples

Example 1: Find δq, where $q = A^2BC$. The values and the uncertainties of A, B and C are known positive numbers.

Solution:

$$q = A^2BC = AABC \quad .$$

Using the "Multiplication and Division" rule, the relative uncertainty in the product is the sum of the relative uncertainties of the constituents:

$$\frac{\delta q}{q} = \frac{\delta A}{A} + \frac{\delta A}{A} + \frac{\delta B}{B} + \frac{\delta C}{C} = 2\frac{\delta A}{A} + \frac{\delta B}{B} + \frac{\delta C}{C} \quad .$$

To find the absolute uncertainty from the relative uncertainty:

$$\delta q = q\left(\frac{\delta q}{q}\right) .$$

Substitute in the numbers to find the numerical value of δq.

Example 2: Find δm, where $m = m_1 + 2m_2$. The values and the uncertainties of m_1 and m_2 are known.

Solution:

Using the "Addition or Subtraction" rule, we add the absolute uncertainties

$$\delta m = \delta m_1 + \delta(2m_2) \quad .$$

Using the "Multiplication and Division" rule

$$\frac{\delta(2m_2)}{2m_2} = \frac{\delta 2}{2} + \frac{\delta m_2}{m_2} = \frac{\delta m_2}{m_2} \quad \text{(because } \delta 2=0\text{)},$$

so $\delta(2m_2) = \dfrac{\delta m_2}{m_2}\left(2m_2\right) = 2\delta m_2$,

and finally $\delta m = \delta m_1 + \delta(2m_2) = \delta m_1 + 2\delta m_2$.

Note that we can also get $\delta(2m_2) = 2\delta m_2$ directly by using the "Multiplication by a Constant" rule.

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Example 3: The mass of a cylinder is measured to be $m = (112.3 \pm 0.1)$ g. The diameter is measured to be $d = (1.23 \pm 0.01)$ cm and the length is measured to be $l = (5.77 \pm 0.01)$ cm. What is the density ρ of the cylinder and its uncertainty?

<u>Calculations</u>

Convert units:

$m = (112.3 \pm 0.1) \times 10^{-3}$ kg, $d = (1.23 \pm 0.01) \times 10^{-2}$ m, $l = (5.77 \pm 0.01) \times 10^{-2}$ m.

So the density is:

$$\rho = \frac{m}{V} = \frac{m}{\pi r^2 l} = \frac{4m}{\pi d^2 l} = \frac{4\left(112.3\times10^{-3}\,\text{kg}\right)}{\pi\left(1.23\times10^{-2}\,\text{m}\right)^2\left(5.77\times10^{-2}\,\text{m}\right)} = 16380\,\text{kg/m}^3$$

Uncertainty Analysis

$$\frac{\delta\rho}{\rho} = \frac{\delta m}{m} + 2\frac{\delta d}{d} + \frac{\delta l}{l}$$
$$= \frac{0.1\text{g}}{112.3\text{g}} + 2\left(\frac{0.01\text{cm}}{1.23\text{cm}}\right) + \frac{0.01\text{cm}}{5.77\text{cm}}$$
$$= 0.018884$$

This is from the multiplication rule. Note that $\delta\pi/\pi = 0$ and $\delta 4/4 = 0$ because they are exact numbers.

percentage uncertainty $\frac{\delta\rho}{\rho} = (0.018884)(100\%) = 1.8884\% \rightarrow 2\%$

absolute uncertainty $\delta\rho = \rho\left(\frac{\delta\rho}{\rho}\right) = (16380\text{ kg/m}^3)(0.018884) = 309.32\text{ kg/m}^3$

<u>Conclusions</u>

The density of the cylinder, ρ, was found to be $\left(1.63 \pm 0.03\right)\times10^4$ kg/m^3 $\left(\pm 2\%\right)$.

(Note that because the uncertainty $\delta\rho$ is bigger than 10 and we want to keep only 1 sig. fig. for it, we have to use scientific notation to report the final result.)