

Graphing and Simple Harmonic Motion, Part I

Many Physics concepts can be modeled as linear relationships: Newton's 2^{nd} Law ($F = ma$), Hooke's Law ($F = k \Delta x$), Ohm's Law ($V = IR$), etc. When experimental data is collected to test these relationships, physically meaningful constants can be found (mass, spring constant, resistance) from the slope of a graph of the data. This is the basis of the $y = mx+b$ form of a linear equation.

However, not all relationships are linear. In fact, even some "linear" relationships are only linear in a limited range (e.g. not all conductors follow Ohm's Law at very low or very high currents). Examples of common non-linear Physics and Biology include:

- exponential population growth equations: $P = P_0 e^{rt}$
- intensity distance equations (aka inverse square law): $I = P/4\pi r^2$, $F = GM_1M_2/r^2$ etc.
- oscillatory motion: $x = A\sin(\omega t + \theta)$
- kinematics with constant acceleration: $y = y_0 + v_0t + 1/2at^2$
- volume of a sphere: $V = 4/3\pi r^3 = 1/6\pi d^3$

We can plot experimental data for all of these phenomena, but the graphs would NOT be straight lines. Thus a graph would not be all that useful as a calculation tool, but only as a way of visually seeing the relationships as a picture.

In this worksheet/lab, we will work through some ways of analyzing experimental data with graphs, both when a relationship DOES follow a linear equation, and when it might not.

Problem 1: Hooke's Law

Springs are tightly wound coils of wire with a structure that will store potential mechanical energy. When compressed or stretched, the force it exerts is proportional to its change in length. This property of springs is called "Hooke's Law", after the guy who experimentally discovered it.

The equation for Hooke's Law is: $F = k \Delta x$,

where F is the force, either the spring pushing out (for compression) or pulling in (for extension),

∆x is the compression or extension length from the spring's rest position

and *k* is the proportionality constant, called the "spring constant" or "force constant", and has units of force/length, which would be Newtons/meter (N/m) in SI units.

Since the form of Hooke's Law is in the linear form of $y = slope \cdot x + b$, it is easy to recognize that the value of the force constant can be found by plotting force on the vertical axis and extension on the horizontal axis of a graph, and calculating the slope of the resulting best-fit line.

Apparatus:

Fig 1

Data:

Calculations: Draw the graph.

- As you plot your data points, include their uncertainties with error bars.
- Draw your "best fit" and "worst fit" lines.
- The "best fit" line should pass as close as possible to as many of the data points as possible. (It does not necessarily HAVE to go through ANY of the points, nor the origin!)
- The "worst fit" line is the line that has a slope that is as different as possible from the "best fit" line, but still touches all of the boxes formed by the horizontal and vertical error bars.
- Calculate the slopes of your "best fit" and "worst fit" lines, on the graph, using big slope triangles and NOT using data points.
- Remember your units, and a zillion other details. (see Graphing: Advanced document)

See graph "Using Hooke's Law to find spring constant for spring # _____" for slope calculations.

m N cm N $x_{h2} - \Delta x$ $F_{b2} - F_{b2}$ *run* $slope_{best} = \frac{rise}{}$ $_{b2}$ $-\Delta\lambda_b$ $b2 \rightarrow b$ δ_{best} – δ_{run} – Δx_{h2} – Δ = − =(− $=\frac{rise}{1}=\frac{F_{b2}-F_{c2}}{1}$ $($ $)$ $($ $)$ 2 $-\Delta t_{b1}$ 2 I_{b1}

(note: These calculations are to be done ON THE GRAPH. You can summarize them in your report too if you like.)

Uncertainty Analysis:

$$
\delta slope = slope_{best} - slope_{worst}
$$
\n
$$
= \frac{N}{2}
$$

so: $k =$ (do not round anything yet!!!)

m

(note: These calculations would be done in your report.)

Conclusions (now you do appropriate rounding)**:**

The spring constant for spring # ______ was found to be $($ _____ \pm _____) N/m (\pm __%), using Hooke's Law. The reference value for spring $\#$ ______ is _(_______ \pm ______)_ *N/m*, according to the list in T346 or on its tag.

Discussion:

Since the range of the measured and reference values overlap, our values agree.

Other comments and observations: (examples: Does Hooke's Law always apply? How can we make the experiment "better"? Is there a y-intercept on the graph, and what does it mean?)