Name: ________________________________

Partner(s): _____________________________

1125 Section: __________

Desk # _______________

Date: _________________

Conservation of Momentum

Purpose

The purpose of this experiment is to verify the conservation of momentum in two dimensions.

Introduction and Theory

The momentum of a body ($\vec{p}$) is defined as the product of its mass (m) and velocity ($\vec{v}$):

$$\vec{p} = m\vec{v}$$

According to the law of conservation of momentum, when two objects collide, if there are no external forces, then the total momentum of the system remains constant after the collision, or

$$\vec{p}_{\text{system,before}} = \vec{p}_{\text{system,after}}$$

where the system momentum is the vector sum of the momenta of the two objects. In this experiment, we will test to see if the law of conservation of momentum is true.

Apparatus Ramp # ____, two steel spheres, one glass sphere, 4-beam balance, newsprint, carbon paper, meter stick, right-angle triangle ruler.

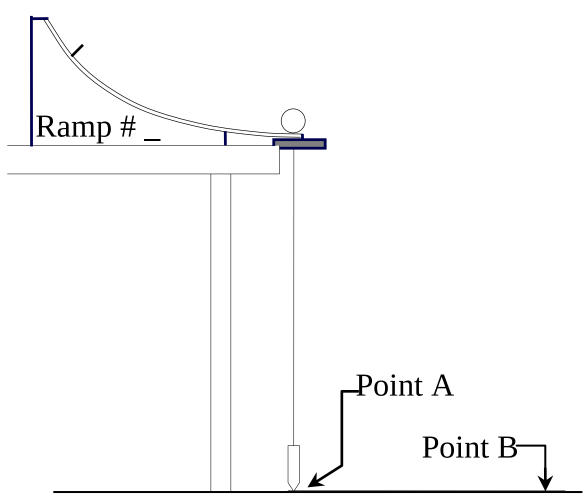


Figure 1

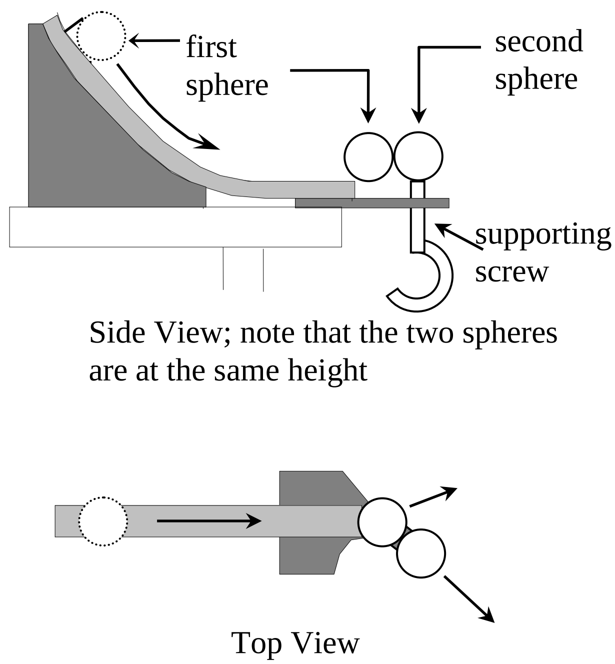


Figure 2

This lab has a different format than others, so let us look at the general picture first. Remember, whenever we mention momentum, we are talking about vectors. Also, keep in mind that momentum is the product of mass and velocity.

We will do two problems in this lab. In Problem 1, we let a steel sphere collide horizontally with another identical sphere at rest, and then let both spheres follow projectile motions to land on the ground. Both spheres start with the same initial height and therefore land on the ground at the same time. (Ignore the difference caused by the uncertainties.) In this case, the momenta can be represented by the horizontal displacement vectors, which can be located by drawing a line from the starting point to the landing point using the same method as in the "Projectile Motions" lab. Adding the momenta of the two spheres, we can get the momentum of the system. By comparing the momentum before and after the collision, we can see whether the momentum is conserved.

In Problem 2, we again let the steel sphere collide horizontally with another sphere at rest, but this time the target is a glass sphere, which is much lighter. The lighter target behaves quite differently: its horizontal displacement easily exceeds in size all other displacements. Is momentum still conserved? This time, we cannot simply use the horizontal displacement to represent the momentum, as the glass sphere has a very large displacement but not so large momentum. However, if we scale down the horizontal displacement of the glass sphere by how much it is lighter than the steel sphere, then we can again use the horizontal displacement vectors to represent the momenta. This of course, requires us to measure the masses of the two spheres. The rest of the analysis is the same as Problem 1.

For each problem, you will perform the experiment, get the vectors on the big newsprint paper, add the vectors on the newsprint paper graphically to obtain the momentum of the system before and after the collision. The only length you need to measure is the length of the horizontal displacement of the glass sphere in Problem 2. The only two calculations you need to perform are also both in Problem 2: (1) calculating the ratio of the masses and (2) calculating the scaled length of the momentum of the glass sphere. Most of your work will be done on the big newsprint paper and make sure you have the instructors sign your newsprint for both problems.

After the experiment, there are two exercises to test if you have mastered the materials of this lab.

Problem 1: Collision between two steel spheres

Setup the apparatus as in Figure 1 and place a sheet of newsprint on the floor beneath the ramp. Do some trial runs by releasing the first steel sphere from the nail position and have it collide off-centred with the second steel sphere sitting on the support screw (see Figure 2). Then let the first sphere roll down alone (with no second sphere).

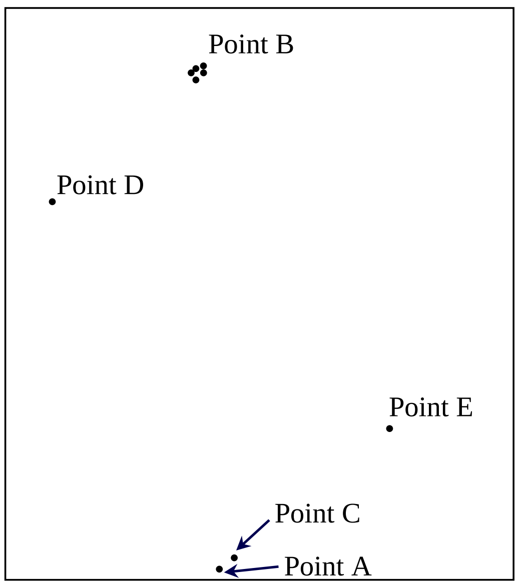
Adjust the orientation of the supporting screw so that the first sphere will not hit the supporting screw without the second sphere present. (But it will hit the second sphere if it is there.) Also check that the two spheres are at the same height when colliding (i.e. the bottom of the track is at the same height as the top of the supporting screw). If they are not, consult the instructor. Do not try to adjust the height of the supporting screw yourself.

Adjust the position of the big newsprint paper so that:
(1) the starting positions of both spheres fall within the big paper;
(2) with the collision, both spheres land on the big paper;
(3) without the collision, the first sphere lands on the big paper.

Once you have achieved the conditions above, keep the screw and the paper in position and do not move them. Secure the position of the newsprint paper on the floor with masses from your mass box.

Create 5 points on the newsprint (Figure 3): the starting positions of both spheres (points A and C), the landing positions of both spheres after they collide (points D and E), and the landing position of the first sphere without the collision (point B). Use a plumb bob to mark points A and C. Points B, D and E are marked by covering the big paper with carbon paper, and allowing the spheres to land on the carbon paper.

To get an idea of uncertainty, mark point B (landing of the first sphere without collision) 5 times. After finishing the data collection, your big paper should look like Figure 3, where point B is a group of 5 points.



Point A – First sphere starting position

Point B – First sphere landing spot without collision

Point C – Second sphere starting position

Point D – First sphere landing spot after collision

Point E – Second sphere landing spot after collision

Figure 3

Now you can remove the big paper from the floor to analyse it. Most of your work will be done on the big paper, but there are also some blanks in this handout that you must fill.

Our goal is to find the vectors that represent the system's momentum before and after the collision. From the knowledge of the "Projectile Motions" lab, if the initial motion of a projectile is horizontal, we can find the initial velocity $\vec{v}$ by the horizontal displacement $\vec{d}$ over the flight time *t*. Therefore, the momentum in the horizontal direction is

$$\vec{p} = m\vec{v} = m\frac{\vec{d}}{t}$$

We will use two letters – from start to finish – representing the horizontal displacement, from example, $\overrightarrow{AB}$ (vector A to B) represents the horizontal displacement of the first steel sphere before the collision. So, $m\frac{\overrightarrow{AB}}{t}$ is the momentum of the first steel sphere before the collision, or $\vec{p}_{\text{steel1,before}} = m\frac{\overrightarrow{AB}}{t}$. Following this format, identify all momentum vectors and fill in the blanks on the next page.

(Note: Strictly speaking, the mass *m* and the flight time *t* may not be the same for the two spheres, but we will ignore this and will discuss it in "Discussions".)

Before the collision,

- The momentum vector of the first steel sphere $\vec{p}_{\text{steel1,before}} =$ ______________.
- The momentum vector of the second steel sphere $\vec{p}_{\text{steel2,before}} =$ ______________.
- The momentum vector of the 2-sphere system is (take the sum) $\vec{p}_{\text{system,before}} =$______________.

After the collision,

- The momentum vector of the first steel sphere $\vec{p}_{\text{steel1,after}} =$ ______________.
- The momentum vector of the second steel sphere $\vec{p}_{\text{steel2,after}} =$ ______________.

To find the momentum vector of the system, we need to factor out *m/t* and then add vectors $\overrightarrow{AD}$ and $\overrightarrow{CE}$. We will add them graphically using the so-called "head-to-tail method" or the "triangle method". Move the second vector so that its tail is at the head of the first vector, point D, and its head is at a new point, let's call it point F. In other words, you move $\overrightarrow{CE}$ to $\overrightarrow{DF}$. $\overrightarrow{AF}$ is the sum of the two vectors, so,

- The momentum vector of the system $\vec{p}_{\text{system,after}} = \frac{m}{t}(____ + ____) =$______________.

(Note: Be careful when moving a vector: *both* the magnitude and the direction must stay unchanged! Using a meter stick and a right triangle ruler, you can move a vector without losing its direction. Still, you must check that points CEFD form a parallelogram.)

To see if vectors $\overrightarrow{AF}$ and $\overrightarrow{AB}$ agree within uncertainties, select one option below:

Point F is ☐ inside circle B ☐ just outside circle B ☐ far from circle B

Ask your instructor to check your work and then sign here: ______________________.

Conclusion of Problem 1:

The momentum of the two-steel-sphere system before the collision is vector ________________, after the collision is vector ________________ . Because ____

____,

these two vectors __________________(agree/disagree) within uncertainties. Therefore, the conservation of momentum is ______________________ (verified/not verified).

Discussions of Problem 1: (In the space below, list the physical factors ignored in this problem.)

Problem 2: Collision between a steel sphere and a glass sphere

Repeat Problem 1, but use the glass sphere on the supporting screw as the target. Use the back of the newsprint to mark the points, and test the drops to ensure all spots land on the paper. Similar to Problem 1, create 5 points on the newsprint: The starting positions of both spheres (points A and C), the landing positions of both spheres after they collide (points D and E), and the landing position of the steel sphere without the collision (point B). Drop the steel sphere alone multiple times to create Circle B. Remove the paper from the floor for analysis.

You must have noticed that point E is much further from the starting point than it was in Problem 1. This is because the glass sphere has a smaller mass. Vector $\overrightarrow{CE}$ has the correct direction of the momentum of the glass sphere, but the length has to be scaled down by the mass ratio in order to represent its momentum, as the following:

1. Measure and record the masses of the spheres, m_{steel} and m_{glass} with a 4-beam balance.

    Mass of the steel sphere: (________________ ± ____________) g

    Mass of the glass sphere: (________________ ± ____________) g

    Mass ratio steel/glass: $q = \dfrac{m_{\text{steel}}}{m_{\text{glass}}} =$ ________________ (Keep 5 non-zero digits)

2. Locate point E′ as the following:

    Length of vector $\overrightarrow{CE}$: ____________ cm (Measure and write it on the big paper and in here.)

    Length of vector $\overrightarrow{CE'}$, which is the above length divided by q. Show the calculation below:

    $\left|\overrightarrow{CE'}\right| = \dfrac{\left|\overrightarrow{CE}\right|}{q} =$ ____

Mark point E′ on the newsprint sheet. The point E′ is where the glass sphere would have landed had it the same mass as the steel sphere. In other words, considering the difference in masses, vector $\overrightarrow{CE'}$ represents the momentum rather than vector $\overrightarrow{CE}$:

$$m_{\text{glass}}\left|\overrightarrow{CE}\right| = m_{\text{steel}}\left(\frac{m_{\text{glass}}}{m_{\text{steel}}}\right)\left|\overrightarrow{CE}\right| = m_{\text{steel}}\left|\overrightarrow{CE'}\right|$$

The rest of Problem 2 is similar to Problem 1, but you must note that the masses of the two spheres are different. First, we write the horizontal momentum vectors using the displacement vectors:

Before the collision,

- The momentum vector of the steel sphere $\vec{p}_{\text{steel,before}} =$ ____________.
- The momentum vector of the glass sphere $\vec{p}_{\text{glass,before}} =$ ____________.
- The momentum vector of the 2-sphere system is $\vec{p}_{\text{system,before}} =$____________.

After the collision,

- The momentum vector of the steel sphere $\vec{p}_{\text{steel,after}} =$ ____________.
- The momentum vector of the glass sphere $\vec{p}_{\text{glass,after}} =$ ____________ = ______________.

For the last two blanks, write the glass sphere's momentum using its own mass first, then rewrite it using the mass of the steel sphere and the scaled-down vector $\overrightarrow{CE'}$. This way, we can factor out $\frac{m_{\text{steel}}}{t}$ and add vectors graphically to find the location of point F on the newsprint. Finally, we can write the momentum of the system after the collision in a form similar to that before the collision:

- The momentum vector of the system

  $\vec{p}_{\text{system,after}} =$ ____.

To see if vectors $\overrightarrow{AF}$ and $\overrightarrow{AB}$ agree within uncertainties, select one option below:

Point F is ☐ inside circle B ☐ just outside circle B ☐ far from circle B

Ask your instructor to check your work and then sign here: ___________________.

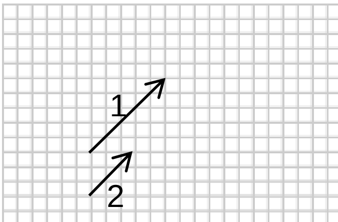
Conclusion of Problem 2:

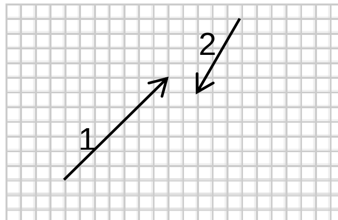
Discussions of Problem 2: (In the space below, list the physical factors ignored in this problem.)

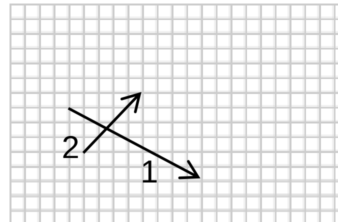
(*Hint: for Problem 1, you may have correctly stated that we ignored the possible small difference between the mass of the two steel spheres. Did you ignore the difference between the masses in Problem 2?*)

Final exercises:

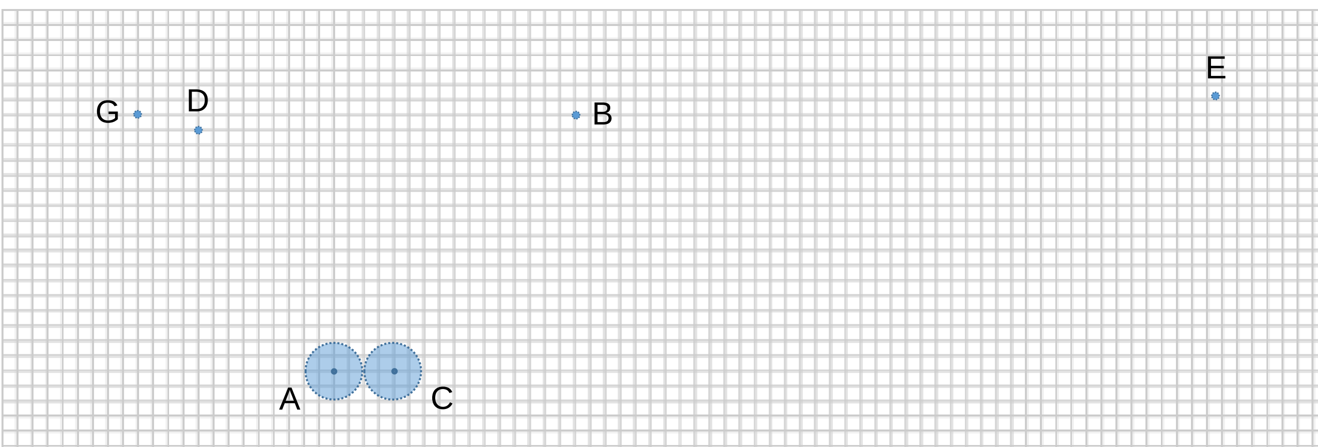
1. Have you mastered the head-to-tail method? For each question, add vectors 1 and 2 to get their sum and mark this vector as $\overrightarrow{AB}$. (Mark A and B on the graph so that $\overrightarrow{AB}$ represents the sum.)







2. Do you understand why we must "scale down the displacement of the glass sphere by the mass ratio"? Two spheres, one steel and one glass, can roll down two separate ramps. If the spheres are released separately, the steel sphere leaves its ramp at point A and lands at point B, and the glass sphere leaves its ramp at point C and lands at point G. When released from the ramps together, the steel sphere and the glass sphere collide horizontally at points A and C, and the steel sphere lands at point D while the glass sphere lands at point E, as shown below. The mass of the steel sphere is 3 times the mass of the glass sphere. Find the system momentum before and after the collision graphically and fill in the blanks below with vectors in the form of $\frac{mass}{time}\overrightarrow{XY}$.
Note: All vectors must be shown on the diagram; this means you will need to name the end of some vectors with letters like F and H. For the glass sphere, write the momentum in its own mass first, then in the mass of the steel sphere, so finally all vectors have the same factors.



Momentum vector of the steel sphere before the collision: _____________.

Momentum vector of the glass sphere before the collision: _____________ = _____________.

Momentum vector of the system before the collision: _____________.

Momentum vector of the steel sphere after the collision: _____________.

Momentum vector of the glass sphere after the collision: _____________ = _____________.

Momentum vector of the system after the collision: _____________.